Sponsored Search :

How Platforms Exacerbate Product Market Concentration*

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How do sponsored advertisements affect product market concentration, through their effects on firms' pricing and consumer behaviour? To analyse this, I develop a theory of digital markets where an intermediary provides a platform for firms to advertise their product and where consumers need to engage in costly search if they want to learn more about the product characteristics. First, I show that when prices are observable prior to the costly product inspection, the less prominent (lower in the search order) firm is forced to lower its price in order to attract more visitors, thus putting it at a competitive disadvantage. Second, I augment this model by allowing the intermediary to determine endogenously, through an auction, the order in which products are displayed and the advertising commissions to be paid. I show that the pass-through from these commissions to product prices is actually higher for the less prominent firm, thus putting it at a further competitive disadvantage. In equilibrium, these asymmetries in consumer price elasticity and commission pass-through lead to lower competition, consumer surplus and total transactions in the product market. Third, I show that the *pay per-click* business model generates higher surplus for the intermediary, compared to the *pay per-sale* and the *consumer subscription fee* models, which improve consumer surplus at the expense of the intermediary's. Fourth, I provide novel empirical evidence that is consistent with some key predictions of the model. These results contribute to the ongoing policy discussions on the effect of dominant digital platforms on product market concentration.

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1 Introduction

The rise of dominant digital platforms has been accompanied by concerns of fall in product market competition across different sectors.[1] This counters initial sentiments that the Internet would facilitate highly competitive markets due to lower search costs and enable more firms to reach consumers.[2] In such online markets, to ensure that they reach as many consumers as possible, firms often compete to advertise their products and to appear in a prominent position. How do these sponsored advertisements (ads) affect firms' pricing and thus, the resulting price competition? Given that platforms' business models frequently rely on the commissions they derive from selling these advertisements,[3] do platforms' commission structure alleviate or exploit consumers' search frictions and what are the implications for overall welfare? As ad revenue plays a major role in monetising a platform's market power, understanding the effect of sponsored ads on product markets could already help us make crucial progress in the pursuit of competitive online markets.

While competition *among* platforms has received extensive attention from policymakers,[4] understanding better the firms' competition *within a* platform can be instrumental in informing effective and comprehensive regulations.[5] Studies on the firms' competition within a dominant platform have largely devoted their attention to the strategic interaction between firms and the platform.[6] However, consumers' frictions remain a crucial understudied determinant of digital market outcomes. In this paper, I study an important device through which a platform affects the online market - Position Auction or Ad Auction - and its interaction with consumer search behaviour, to understand firms' pricing strategies.

I model the platform's ad revenue as a function of an auction that it designs and conducts to determine (i) the order in which ads (of firms) are displayed, and (ii) the ad commission to be paid at each position. I show that this ordered display not only determines consumers' click-through and firms' pricing behaviour, but also endogenously affects product market concentration and

[1]See, for instance, Khan (2018), ACCC (2019), Crémer, de Montjoye, and Schweitzer (2019), Scott-Morton, Bouvier, Ezrachi, Julien, Katz, Kimmelman, Melamed, and Morgenstern (2019) and CMA (2020b) for dominance of platforms. And Gutiérrez and Philippon (2017), Grullon, Larkin, and Michaely (2019) , Autor, Dorn, Katz, Patterson, and Van Reenen (2020), De Loecker, Eeckhout, and Unger (2020), Tambe, Hitt, Rock, and Brynjolfsson (2020) and Affeldt, Duso, Gugler, and Piechucka (2021) for rise in concentration.

[2]See, for instance, Brynjolfsson and Smith (2000) and Brynjolfsson, Hu, and Smith (2003) for some early empirical evidence of this sentiment and Pozzi (2012) for some later counter-evidence.

[3]For instance, Google ad revenue (see http://investor.google.com/) accounts for more than 80% of Alphabet's total revenue and Amazon reports (see https://ir.aboutamazon.com/) more than 40% annual growth in ad revenue (in comparison, Amazon's total revenue grew by 33.5%).

[4]See, for instance, Broadbent (2020) for an overview of the Digital Markets Act and the Digital Services Act.

[5]See, for instance, Caffarra, Etro, Latham, and Scott-Morton (2020) for a policy discussion on within-platform competition.

[6]See, for instance, Belleflamme and Peitz (2019a) and Teh (2022) for recent academic work on within-platform competition.

platform revenue. Specifically, I highlight a novel channel - the asymmetry, across ad positions, in pass-through from ad commissions to product prices - which leads to an increase in product market concentration and a decrease in consumer surplus. I also provide novel empirical evidence, using data from an anonymous digital platform in the US, that provides support to this channel. This channel shows that even when there is no friction in access to product prices, and even when the intermediary is not directly competing with firms, there can be an exacerbation of market concentration.

I build a model of the e-commerce environment comprising of three key features. First, information on product prices is more easily accessible to consumers than product characteristics.[7] For example, when consumers see a list of ads, they observe the prices of the products on the ads immediately. This information is obtained at a relatively much lower cost compared to how they have to *search* (click on each ad) to learn the *match value* (from other product characteristics/attributes) of the products.[8]

Second, prominence plays an important role in determining consumer behaviour.[9] For example, consumers may start their search from a firm because its ad occupies the top position, a larger banner space or earlier spot than others.[10] Another example could be that of a platform that provides an affiliated brand a prominent position in the market, through display characteristics, ease of access, etc.[11] In equilibrium, I endogenise product prominence and also show that the above-described consumer behaviour arises endogenously, consistent with the traditional rationale of prominence where consumers start their search from the lower-priced firm.

Third, a platform's business model of ad sales is captured using a Generalised Second Price auction which is conducted by the platform to maximise its revenue.[12] The platform's revenue

[7] See Figure A.1 for some examples. See, for instance, Gorodnichenko and Talavera (2017) for a discussion on the ease of access to prices online due to the ubiquity of price-comparison websites.

[8] As consumers pay a uniform search cost in my model, any differences in costly-to-observe "add-on" prices are inconsequential in equilibrium, following the Chicago-style arguments (Lal and Matutes, 1994). Moreover, add-ons (e.g., shipping cost) are often used to recover market-specific (e.g., product/location-based) fixed costs (Ellison, 2005) and are independent of total visitors. Therefore, one could also consider "add-ons" as a part of the match value, in my model, that is discovered after search. Hence, my framework allows me to focus on the firms' choice of the advertised product price.

[9] See, for instance, Granka, Joachims, and Gay (2004) for eye-tracking evidence, Ghose and Yang (2009) for evidence from panel data at Google, Agarwal, Hosanagar, and Smith (2011) for evidence from a field experiment at Google, Narayanan and Kalyanam (2015) for evidence from an RDD approach, Ursu (2018) for evidence from a field experiment at Expedia, Simonov and Hill (2021) for quasi-experimental evidence from Bing, and Moshary (2021) for evidence from a field experiment.

[10] See Figure A.1 for some examples. Although I will use the context of search ads to describe my model, the insights can be interpreted in other contexts of advertising (e.g., display ads and video ads) and 'product' markets (e.g., finance: insurance ads, labour: job search ads) with some qualifications.

[11] See, for instance, https://support.google.com/google-ads/answer/6381002?hl=en for Google's definition of prominence.

[12] This is one of the most common auction formats employed by digital platforms to determine ad positions. See, for instance, Edelman, Ostrovsky, and Schwarz (2007) and Varian (2007).

equals 'pay per-click' commission times the number of clicks or visitors to each firm.[13] This auction, in turn, determines the order of firms displayed to the consumers and the ad commission to be paid by firms, which is a function of their position and market competition.

Using this framework, I determine the endogenous search and demand behaviour of consumers over products (or ads, in this paper), as well as the pricing decisions of firms (for their product) and the platform (for its ad position). A product's price is determined by both consumers' preference and the pass-through from the commission paid by the firm to the platform. In turn, a firm's commission is a function of its position on the list. As a result, the number of clicks a firm gets and the cost per-click vary with position. These differences in cost, and their effect on product pricing, allows me to micro-found the 'value of a click' based on position and firm characteristics, thus avoiding the common assumption of position homogeneity.[14]

First, I analyse the market outcomes in the absence of an auction. For this exercise, I assume that consumers start their search from the prominent firm, in order to highlight the effect of prominence and price visibility on product pricing and competition. I show that, when a pure-strategy equilibrium exists, firm 1 (in prominent position) charges a higher price than firm 2 (in non-prominent position), that is $p_1 > p_2$. Further, this price dispersion increases with search cost. These results are in contrast with the standard result from the literature on prominence where firms' prices increase with order and converge at high search costs (see, for instance, Armstrong, Vickers, and Zhou, 2009). When prices are free to observe (hereafter, *in-sight* prices), a lower p_2 not only retains more visitors, but is also likely to attract more of them. The latter effect makes firm 2's demand more elastic and puts it at a competitive disadvantage.

Second, I introduce endogenous ad cost (or commission) for firms. The platform chooses the auction reserve price such that its revenue is maximised, which then determines the commissions and the order of firms. The platform's choice, in turn, is governed by the participation and incentive constraints of the two firms. In this analysis of the full equilibrium, I allow consumers to choose their search order. I show that there exist search costs for which prices are increasing with order ($p_1 < p_2$). This result reconciles with the rationale of prominence, from previous work on directed search, where consumers indeed start their search from the lower-priced prominent firm. However, the order in my model is driven by the ad commission structure. A duopoly model helps me clearly contrast the asymmetric effects of ad commission on a prominent versus non-

[13]This formulation of platform revenue is motivated by Google ranking firms based on PPC times the estimated click-through rate. See https://support.google.com/google-ads/answer/142918?hl=en (Accessed May 14, 2020) - "The most important thing to remember is that even if your competition bids higher than you, you can still win a higher position – at a lower price – with highly relevant keywords and ads."

[14]See, for instance, Goldman and Rao (2016) for evidence from Bing on heterogeneous effect of position on click-through rates.

prominent firm. Specifically, commission paid by the prominent firm is analogous to a fixed cost since the number of visitors it gets is constant with respect to its price. However, commission is analogous to marginal cost for the non-prominent firm, since how many clicks its ad gets depends on the price that consumers see before clicking. This cost structure asymmetrically forces firm 2 to raise p_2 to compensate for the rise in its 'marginal cost', thus putting firm 2 at a further competitive disadvantage.

A complementary intuition for the above result can be developed using the *conversion rate* (ratio of buyers to visitors). This ratio is determined by the price elasticity of consumers that a firm faces. In a standard model of prominence, since the prominent firm faces more price elastic consumers ("fresh" consumers, as referred to in the literature), it charges a lower price in equilibrium and "self-fulfils" by attracting consumers to visit it first. Hence, the fraction of "fresh" consumers faced by a firm appears to be a property of its position. In my paper, I study the online ad environment, where firms compete for such ad positions, and where their product prices are visible prior to search. In this setting, the intuition from the standard models breaks down, since part of that mechanism (holdup) is eliminated by price visibility, and the order of prices is no longer "self-fulfilling". However, once we take into account the ad auction, the self-fulfilling equilibrium is recovered. This setting also endogenises position allocation. Intuitively, firms here are actually competing to get a conversion rate that will give them a lower pass-through of ad commissions. This cost-side mechanism is what completes the circle and generates equilibrium prices that "self-fulfil" prominence. A comparative statics exercise with search cost in Section 4 provides falsifiable predictions on conversion rate and product pricing, which are then tested empirically in Section 7.

As an extension, I introduce an additional dimension of product heterogeneity due to vertical differentiation. This exercise highlights situations where consumers can benefit from platform recommendations. However, note that the preferences of consumers, firms and the platform may not be well-aligned. For instance, since the total commission depends on the number of searches, a platform can place a firm of lower quality or a firm of lower relevance at the first position, thus inducing more searches. This could be suboptimal for the consumers and firms. Results from section 5 suggest that this is not the case. A firm that is of higher quality or of more relevance to a user stands to earn higher in the first position, and therefore, promises higher commission-per-click to the platform and procures the prominent position.[15]

Third, I analyse the welfare implications under alternative intermediary revenue models. In this endeavour, first, I study the (i) *pay per-sale* and (ii) *consumer subscription fee* models, in place of the *pay per-click* model, and show that they can improve consumer welfare at the expense of

[15] Note that this need not be the case when prices are *out-of-sight*. See, for instance, Jerath, Ma, Park, and Srinivasan (2011).

Table 1: Evidence on prominence and search order

Unconditional:	
$\mathbb{P}$[receiving a click if prominent]	0.36
$\mathbb{P}$[receiving a click if non-prominent]	0.10
$\mathbb{P}$[receiving a click if non-prominent $\mid$ prominent ad was clicked]	0.27
$\mathbb{P}$[receiving a click if non-prominent $\mid$ prominent ad was not clicked]	0
Within ad:	
$\mathbb{P}$[receiving a click if prominent] - $\mathbb{P}$[receiving a click if non-prominent]	0.07
	(0.0001)
Total observations	1,612,594

the intermediary. Second, I study a combined model of *pay per-click* and *consumer subscription fee* and show that the intermediary prefers to cross-subsidise consumers, while facing a trade-off between reducing revenue from ad commissions and collecting fees from consumers. Third, I study the Generalised First Price auction, leaving the *pay per-click* design unchanged, and show that the outcomes coincide with that of the GSP auction, for the case where two firms are competing for the ad positions.

Finally, I use clickstream data from an anonymous platform in the US, to test some key predictions of the model. Using a novel combination of information on ad commissions, ads shown to users and user click & purchase behaviour, I test the theoretical predictions at the platform-optimal level of search cost. Table 1 shows that prominent ads receive more clicks and that consumers' search order follows the ad display order. Figure 7 provides a further description of the main empirical variables, by comparing an ad that is displayed in a prominent position with the same ad when it is displayed in a non-prominent position.

I use a selection on observables approach to control for ad characteristics, quality, type of product, time of the day and day of the week. Consistent with the model, I find that an ad in the prominent position, compared to a similar ad in a non-prominent position, (i) pays higher commission per-click, (ii) receives more clicks, (iii) converts a similar fraction of clickers, (iv) charges a similar price, and (v) receives the first click from a user session. This evidence provides support to the mechanism illustrated in the theoretical framework: the conversion rate and hence, variation in firms' ad cost pass-through across positions, plays a key role in determining the variation in firms' product price across positions.

1.1 Literature Review

This paper contributes to three broad strands of literature, namely those of firm-intermediary interaction and ordered consumer search, and advertising.[16,17]

Firm-intermediary interaction. Edelman, Ostrovsky, and Schwarz (2007) and Varian (2007) explore the question of how firms bid to place themselves at their preferred position. They approach the problem from the perspective of optimal auctions. My paper differs in terms of both motivation and modelling. They omit the analysis of pricing behaviour, while my main interest lies in the competition structure resulting from the pricing behaviour. De Corniere and Taylor (2016) studies the effects of collusion (bias) between an intermediary and a firm. Hagiu and Jullien (2011) studies the incentives for an intermediary to divert search when contracts are exogenous between intermediaries and firms. In contrast, my paper explores how an *endogenous bias* can emerge within a competitive setting, even when the intermediary itself is not inherently biased towards a particular firm. Inderst and Ottaviani (2012) studies the role of commissions in shaping intermediary advice using Hotelling's framework. My model instead embeds ordered search in position auctions, which enables me to microfound the heterogeneous value of clicks at each ad position. This provides a more granular understanding of how ad commissions translate into competitive advantages.

Recent work by Anderson and Renault (2021) also investigates the impact of ad auctions on firms and consumers. However, their model primarily focuses on demand-side externalities arising from ad position, with product prices not directly influencing clicks or bids. My paper introduces a crucial mechanism: the pass-through of ad commissions to product prices, demonstrating how this channel significantly alters market outcomes. Similarly, Kang (2021) examines ad auctions, but his analysis centers on first-price auctions. In contrast, my paper analyzes a second-price auction, which allows for a richer interaction where a competitor's product pricing directly influences a firm's bidding strategy. Motta and Penta (2022)'s approach is complementary to mine as it builds on asymmetries in products due to consumers' prior interest (observed, for instance, when consumers browse for brand-specific keywords) to show that ad auction favours the brand that a consumer is interested in prior to search, over its competitor. My paper, though, captures consumer behaviour using a sequential search model

[16] Academic work on *hybrid* platforms also analyses competition *within a* platform, focusing on a different set of trade-offs (largely focused on self-preferencing by intermediary) from this paper. See, for instance, Hagiu, Teh, and Wright (2020), Anderson and Bedre-Defolie (2021) and Hervas-Drane and Shelegia (2021).

[17] The literature on multi-homing also explores the strategic interactions between consumers and platforms, focusing on a different set of trade-offs (largely focused on demand-side network externalities) from this paper. See, for instance, Doganoglu and Wright (2006) and Belleflamme and Peitz (2019b). See Jullien and Sand-Zantman (2021) for a recent survey on the effect of network externalities on consumers and platform competition. See Bergemann and Bonatti (2019) for a recent survey on platforms as information intermediaries.

over symmetric products (observed, for instance, when consumers browse for generic keywords), which also allows me to speak closely to the literature on ordered search and prominence. Another difference is that my model focuses on situations where firms do not commit to product prices before setting their bids.[18]

Ordered consumer search. Haan, Moraga-González, and Petrikaitė (2018) and Choi, Dai, and Kim (2018) study a directed search model where product prices determine consumers' path. In contrast, my paper takes a step back, and considers an intermediary that determines the price-path endogenously, which then affects the consumers. This allows me to incorporate the incentives of a major player, the platform. Chen and He (2011) studies a model with identical products and exogenous matching probability, and hence, abstracts from the pricing issues that arise in my framework. Similarly, Athey and Ellison (2011) also incorporates consumers but focuses on the welfare implications of incomplete information in different auctions and does not address price competition. In contrast, I show that prominence systematically distorts market prices through asymmetric commission pass-through, a mechanism previous models cannot capture.

Armstrong and Zhou (2011) considers the role of per-sale or lump-sum commission payments, which inflate a supplier's marginal cost, and hence, can inefficiently drive up retail prices. In contrast, my paper studies firms pay commissions per-click received, and highlights how the inflation in supplier's marginal cost can be asymmetric across ad position even for symmetric firms. Further, I compare the welfare across the different methods of commission payments and discuss the implications of each for consumers and the intermediary.

Advertising. While Sahni and Zhang (2019) and Moshary (2021) emphasize how ads expand consideration sets (informative role), I reveal how prominent positions restrict them (persuasive role) through price mechanisms that make non-prominent firms systematically less competitive.[19] My paper shows that by obtaining the more prominent sponsored position, a firm can restrict consumers' consideration set.

The remainder of this paper is organized as follows: Section 2 presents the key features of the model. Section 3 analyses consumer demand and firm pricing in the absence of an auction. Section 4 augments this framework with an intermediary (platform) that endogenises firm

[18] See, for instance, Gorodnichenko and Talavera (2017) for recent evidence on the high frequency of price changes in online markets. Also, for example, in Expedia: pay per-click model, firms set their preferences for commissions at a daily-level but are free to change prices during the day. Therefore, the assumption on timing can also be interpreted as firms anticipating their equilibrium prospects of revenue, and setting their auction budget in advance. This is in line with the interpretation provided by Edelman, Ostrovsky, and Schwarz (2007) and Varian (2007), where the static auction is used as a long-term approximation of the highly dynamic ad auction environment, and hence, firms may be treated as being capable of anticipating outcomes.

[19] See, for instance, Clark (2007), Janssen and Non (2009), Rauch (2013) for some earlier work on informative vs persuasive advertising.

positions and commissions, and derives the equilibrium outcomes. Section 5 extends the model to the case of asymmetric firms. Section 6 considers some counterfactual scenarios and compares the welfare implications. Section 7 introduces the data and discusses the empirical evidence. Section 8 concludes by discussing some avenues for future research. Proofs are provided in the appendix.

2 Model

Consider a market that consists of one intermediary (m), two firms (i, j) and a unit mass of consumers. Each firm produces one product and is interested in posting one ad through the intermediary.[20] Firms and the intermediary maximise their own profits, while consumers maximise their utility. Numeric subscripts denote the position of the firm: 1 denotes the prominent position and 2 denotes the non-prominent.

The intermediary conducts a Generalised Second Price (GSP) auction to determine the order in which firms will be listed on its platform. It chooses the reserve price of the auction, denoted by $\widehat{b} \in \mathbb{R}_+$, such that its revenue is maximised. The revenue that it generates equals per-click commission for each firm times the number of clicks (visitors) the respective firm gets. I assume that the platform is unaware of consumers' individual product valuations a priori.[21]

Firms compete with each other for the position in which they are displayed to consumers. Firms bid the per-click commission they are willing to pay and are, in turn, displayed (ordered) based on the outcome of the GSP auction. Each firm produces one product and sets its price after its position is known (see Section 4.4 for discussion on timing). Let $b_i,\ b_j \in \mathbb{R}_+$ denote the bids and $p_i,\ p_j \in \mathbb{R}_+$ denote the prices that each firm sets. Let $r_i(b_i, b_j, \widehat{b}),\ r_j(b_i, b_j, \widehat{b}) \in \mathbb{R}_+$ be the ad cost or commission per click, paid by each firm to the intermediary. Note that the bids are an input to the auction while the per-click commissions are the outputs. To simplify the analysis, the marginal cost of production for both firms is assumed to be zero.

Consumers face a search cost $s \in \mathbb{R}_+$. As widely observed on e-commerce platforms, I assume that they observe the order of listed firms and the prices of each product without paying the search cost.[22] However, they discover their idiosyncratic valuation of a product only after they

[20] Therefore, I will also use the terms 'firm' and 'product' to refer to its 'ad'.

[21] This assumption allows me to clearly interpret the market under study. Each auction conducted by the intermediary gives firms access to consumers of a certain type (e.g., demography, characteristics, search history, time of access). Therefore, the intermediary in my model has already "targeted" its recommendations, given the information it has, and cannot customise the order further for each consumer within this type. However, there remains some heterogeneity within this population of users that the platform is unaware of, which is captured in the model below using v_1, v_2.

[22] I compare my results with a benchmark model where consumers do not observe prices prior to paying the search cost. I find that free price visibility plays a crucial role in understanding demand and pricing (see Online Appendix - Section A).

visit the seller's page, thereby paying the search cost. As an example, imagine a website where consumers see the price on each ad on the homepage but one has to click on it and visit the product-specific webpage to learn about its features (see Figure A.1 for some examples).

Let $v_i,\ v_j$ be a consumer's idiosyncratic valuations of firms (or products) i and j respectively. The outside option for consumers is normalized to 0 and recall is assumed to be costless. Consumers draw their valuations for products sold by firms i, j from an identical distribution F.

$$v_i, v_j \sim F[\underline{v}, \overline{v}]$$

F is twice differentiable and $f(\cdot) > 0$ in this domain. Consumer utility for product i is given by $u(p_i) = v_i - p_i$. Firms and the intermediary are also assumed to be risk neutral and to have an outside option of 0.

The order of events is as follows: First, the intermediary sets the auction reserve price. Second, firms place their bids. Third, the position of firms are revealed. Fourth, firms set product prices. Finally, consumers make search and purchase decisions. I use the solution concept of Sub-game Perfect Nash Equilibrium.

3 Consumer and Firm Behaviour

I begin the analysis by focusing on the effect of consumer search frictions on firms' pricing strategies in the absence of commissions, taking the order of firms as exogenous. In Section 4, I introduce the role of intermediary, which endogenises the position of a firm and its commission cost.

3.1 Consumer Search

To capture the notion of prominence in a simple yet stark manner, I assume that the first search (click) is free for consumers, but a search cost of $s > 0$ applies for visiting firm 2. This induces consumers to start their search from the prominent firm. Note that this assumption does not affect my main result (see Section 4.4 for a discussion).[23]

Consumers know their (indirect) utility from product 1 immediately as they begin to search.

$$u(p_1) \ = \ v_1 - p_1$$

Then, the only search decision that consumers have to make is whether to visit firm 2. The

[23] Note that consumer search frameworks often treat the first search as free for consumers. I also provide empirical evidence in Section 7 that supports the assumption that consumers search by the order displayed.

expected gains of visiting firm 2 are

$$l(v_1, s, \mathbf{p}) \;=\; \int_{v_1-p_1+p_2}^{\overline{v}} (v_2 - p_2 - \max\{v_1 - p_1, 0\})\; f(v_2)\; dv_2 \;-\; s \tag{1}$$

I define the reservation value, $\widehat{v}(s, \mathbf{p})$, as the solution to $l(v_1 = \widehat{v}, s, \mathbf{p}) = 0$. It measures the value of opportunity cost that a consumer has to forego if they decide to purchase the offer in hand. A rational consumer determines this value by taking into account all available information at the time of making the search decision. Hence, prices have a direct effect on search. This is in contrast with the case of *out-of-sight* prices where consumers base their search decision on their expectation of p_2.[24]

Consumers buy at firm 1 without visiting firm 2 whenever their offer in-hand exceeds their reservation value, $v_1 > \widehat{v}(v_1, s, \mathbf{p})$. Otherwise, they visit firm 2. For brevity, I write $\widehat{v}(v_1, s, \mathbf{p})$ as $\widehat{v}$. The price difference is denoted by $\Delta p = p_1 - p_2$.

Lemma 1 (Reservation value, $\widehat{v}$). *The market's reservation value increases with search cost and with price of product 1, but decreases with price of product 2.*

I illustrate the results (throughout the paper) using $F = U[0, 1]$ as a working example. The explicit form for reservation value is then given by[25]

$$\widehat{v} \;=\; 1 + \Delta p \;-\; \sqrt{2s}$$

where prices play a direct role in the search decision.

Some consumers realise a value at firm 1 such that $v_1 - p_1 < 0$. These are consumers who prefer the outside option to product 1. If at all they visit firm 2, they would only compare its product with the outside option value of zero. Since their incentives to search are slightly different, it is determined by whether

$$\int_{p_2}^{\overline{v}} [(v_2 - p_2)] f(v_2) dv_2 \;\geq\; s$$

Let

$$A := \mathbb{1}\{\mathbb{E}[v_2 | p_2 \leq v_2 \leq \overline{v}] \;-\; p_2 \cdot [1 - F(p_2)] \;\geq\; s\} \tag{2}$$

[24]When prices are not free to observe prior to costly search, even though consumers' expectations are true in equilibrium, any deviation in firm 2's price will not affect the number of its visitors (that is those consumers who decide to search).

[25]Note that the expression for the case of *out-of-sight* prices is $\widehat{v} = 1 - \sqrt{2s}$.

denote the “Attraction” conditions under which consumers who realised $v_1 < p_1$ will continue to search and visit firm 2. Intuitively, when this constraint binds, it imposes an upper bound on the price that the second firm can charge if it desires to attract those who discovered a low valuation at firm 1. For $F = U[0,1]$, this condition becomes $p_2 < 1 - \sqrt{2s}$. Let me denote the region where this constraint binds by $s \geq \underline{s}$.[26]

There are two forces that determine $\widehat{v}$. First, there is the first-order effect of consumer search friction, where $\widehat{v}$ is decreasing in s. When the search cost is high, the opportunity cost of staying put is lower. Hence, the reservation value is lower and the consumer stops searching for lower draws of v_1. Secondly, there is another force due to *in-sight* prices, where the reservation value is increasing in the difference between prices (Δp). Relatively cheaper the first firm is, the sooner a consumer stops searching. This applies a downward force on product prices by inducing an additional competitive element in attracting visitors, and not just retaining them. This second force plays a key role in determining consumers’ decision and subsequently, firms’ pricing.

Search rule. Combining the above steps, we can characterise search behaviour. For consumers who find product 1 affordable ($v_1 > p_1$), one would continue searching only if $v_1 < \widehat{v}$. For consumers who find product 1 unaffodable ($v_1 < p_1$), their decision is determined by the “Attraction” condition in Equation 2. When both conditions fail, consumers exit the market. This rule is similar to the classic result in Wolinsky (1986).

Demand. I denote by D_{xy} the demand for firm x among consumers who visit y number of firms.[27] For example, D_{12} denotes those individuals who visit both firms but finally purchase from firm 1. Thus, demand for firm 1 is given by $D_1 = D_{11} + D_{12}$ and demand for firm 2 is given by $D_2 = D_{22}$.

D_{11} denotes the consumers who do not continue searching. They discover their valuation of product 1 such that $v_1 > \widehat{v}$. Hence,

$$D_{11} = 1 - F(\widehat{v}) \tag{3}$$

D_{12} consumers satisfy two conditions. They want to search ($v_1 < \widehat{v}$), but prefer the first firm after discovering their valuation of product 2 ($v_1 - p_1 > v_2 - p_2$). Hence,

$$D_{12} = \int_{p_1}^{\widehat{v}} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1 \tag{4}$$

[26] For a general F, this threshold can be expressed as

$$\underline{s} = \{t | \ \mathbb{E}[v_2 | p_2 \leq v_2 \leq \overline{v}] - p_2 \cdot [1 - F(p_2)] > s \ \forall s < t\}$$

[27] For brevity, I write $D_{xy}(\mathbf{p}, \mathbf{b}, \widehat{v}(s, \mathbf{p}), \widehat{b})$ as D_{xy} throughout the paper.

D_{22} consumers also satisfy two conditions. They also want to search ($v_1 < \widehat{v}$), but prefer the second firm after discovering their valuation of product 2 ($v_1 - p_1 < v_2 - p_2$). Additionally, D_{22} also includes consumers who prefer the outside option of zero over product 1 ($v_1 < p_1$) and continue to search, but prefer the second firm over the outside option after discovering their valuation of product 2 ($v_2 > p_2$). Hence,

$$D_{22} = A \cdot [1 - F(p_2)]F(p_1) + \int_{p_1}^{\widehat{v}} [1 - F(v_1 - \Delta p)] \cdot f(v_1) \cdot dv_1 \tag{5}$$

where A denotes the "Attraction" condition from Equation 2.

Following the terminology commonly used in the literature on prominence, D_{11} and D_{22} are also referred to as *fresh demand* since these buyers are visiting the respective firms for the first time. D_{12} is also referred to as *returning demand* since they are visiting the firm for the second time. Each of these demands are driven by different combinations of forces. This asymmetry in firms' demand invites careful consideration and, in fact, turns out to be the key element behind the equilibrium outcome derived in Theorem 1. Lemma 2 shows comparative statics of these demands with respect to search cost.

Lemma 2. *In the region where prices are not subject to the "Attraction" condition (low search costs), fresh demand for firm 1 is increasing with search cost* $\left(\frac{\partial D_{11}}{\partial s} > 0\right)$*, returning demand for firm 1 is decreasing with search cost* $\left(\frac{\partial D_{12}}{\partial s} < 0\right)$*, and demand for firm 2 is decreasing with search cost* $\left(\frac{\partial D_{22}}{\partial s} < 0\right)$.

An increase in search cost leads to a decrease in the reservation value. Therefore, fewer consumers search, thus, implying a rise in D_{11}. This, in turn, reduces the number of visitors to firm 2 and exerts a downward force on D_{12} and D_{22}. For consumers who still decide to search, we can deduce that they had a low valuation at firm 1. This would have a negative impact on firm 1's returning demand but a positive impact on firm 2's demand. The two-fold negative force lowers D_{12}. For D_{22}, the above two forces act in opposite directions. Overall, we see that the first effect dominates in the region where firm 2 can adjust its price freely (when Equation (2) does not bind).

3.2 Firms' problem

Baseline. Firms 1 and 2 set prices to maximise their revenue. This exercise highlights the effect of prominence and price visibility on firms' pricing in the absence of the intermediary.[28] Firms'

[28] One can also interpret the objective functions as profit functions representing the case of offline advertising where firms pay a fixed cost for prominence. See, for instance, Hristakeva and Mortimer (2021) for an overview of television ads, a major component of advertising expenditure in the pre-digital era.

revenues are given by

$$\begin{aligned} \max_{p_{1,NA}} \; Rev_1 &= p_{1,NA}(D_{11} + D_{12}) \\ \max_{p_{2,NA}} \; Rev_2 &= p_{2,NA} D_{22} \end{aligned} \tag{6}$$

I use the subscript NA to denote the case of 'No Auction'. I use the following benchmark prices to compare with: price set by a monopoly firm (p^{Mon}) and symmetric price set in the case of random search with two firms (p^{Ran}).[29]

3.3 Equilibrium

Using inverse demand functions from Equations (3), (4) and (5), I optimise the objective functions in Equation (6) simultaneously, to obtain equilibrium prices. Lemma 3 shows that even when firms are symmetric in terms of their marginal cost and distribution of product match values among consumers, there is no symmetric pure-strategy equilibrium in prices.

Lemma 3. *In the benchmark case without auction, for a positive search cost, there exists no symmetric pure-strategy equilibrium in firms' product prices.*

This result is driven by the asymmetry in composition of demand due to their respective positions. To further understand the effect of this asymmetry, I solve for the asymmetric equilibrium in pure strategies. Theorem 1 characterises the equilibrium prices.

Theorem 1 (Prices, without auction)**.** *In the benchmark case without auction, for a positive search cost, there is a unique asymmetric pure-strategy equilibrium in firms' product prices. In particular, the prices are given by the following implicit functions.*

$$p^*_{1,NA} = \frac{1 - F(\widehat{v}^*) + \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1}{f(\widehat{v}^*) - \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1}$$

$$p^*_{2,NA} = \frac{A \cdot [1 - F(p^*_{2,NA})]F(p^*_{1,NA}) + F(\widehat{v}^*) - F(p^*_{1,NA}) - \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1}{A \cdot f(p^*_{2,NA})F(p^*_{1,NA}) + f(\widehat{v}^*) - F(p^*_{2,NA})f(p^*_{1,NA}) - \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1}$$

The intuition behind the expression for prices in Theorem 1 is as follows. Consider the unit mass of consumers plotted in Figure A.1. The co-ordinates of a point (v_1, v_2) in the square denote a particular consumer-type with those match values. In this way, this plot helps organise the

[29] The benchmark prices are given by the expressions

$$p^{Mon} = \frac{1 - F(p^{Mon})}{f(p^{Mon})} \qquad , \qquad p^{Ran} = \frac{1 - F(p^{Ran})^2}{F(p^{Ran}) \cdot f(p^{Ran})}$$

heterogeneity in the market and illustrates the objective for each firm. For a small change in p_2, the demand for firm 2 changes along both segments LM and MN. Since firm 1 also shares segment LM, its demand from consumers in this region would be affected symmetrically by a change in p_1. The asymmetry between the firms arises due to segment MN, which represents those consumers who decide to not visit firm 2 because of a change in p_2. Firm 1 does not face such a trade-off, hence putting firm 2 at a competitive disadvantage. To summarise, the price of firm 2 plays a dual role of attracting visitors and converting visitors into buyers. This asymmetry in demand elasticity determines the asymmetry in equilibrium prices.[30] To further illustrate the intuitions behind the above result, I make use of my working example of $F = U[0, 1]$.

Proposition 1. *When match values are drawn from a Uniform distribution ($v \sim F = U[0, 1]$), the reservation value is given by $\widehat{v} = 1 + \Delta p - \sqrt{2s}$ and*

- *the search cost at which the "Attraction" condition starts binding is given by $\underline{s}_{NA} = \frac{1}{4}$, and the search cost at which firm 2 drops out of the market is given by $\overline{s}_{NA} = \frac{1}{2}$,*
- *the price of prominent firm increases with search cost $\left(\frac{dp^*_{1,NA}}{ds} > 0\right)$, and the price of non-prominent firm decreases with search cost $\left(\frac{dp^*_{2,NA}}{ds} < 0\right)$,*
- *and the price difference is positive: $p^*_{1,NA} > p^*_{2,NA}$*

Corollary 1. *The price of the prominent firm lies between p^{Ran} (at $s = 0$) and monopoly price (at $s = \overline{s}$), while the price of the non-prominent firm lies between p^{Ran} (at $s = 0$) and 0 (at $s = \overline{s}$).*

$$p^*_{1,NA} \in [p^{Ran}, p^{Mon}] \qquad , \qquad p^*_{2,NA} \in [0, p^{Ran}]$$

Figure 2 summarises the results from Proposition 1 and Corollary 1. At zero search cost, the model collapses to the case of Perloff and Salop (1985). Thus, we have the full information price as the equilibrium. As search cost increases, fewer consumers search.[31] This means that competition reduces gradually with s, in region I. This can be seen from the increase in the price of firm 1. One would expect a rise in the price of firm 2 as well. However, since prices are *in-sight*, it deters firm 2 from deviating to a higher price as it would immediately lose visitors (who are potential buyers). For firm 2, this loss is larger than the benefit from deviating, and charging a high price to a selection of "high-interest" visitors who, despite all the above frictions, come because they have a relatively larger interest in product 2. Note that the fraction of high-interest visitors increases with an increase in search friction, but on the other hand, the loss of other visitors also

[30]To further understand effect of *in-sight* prices, I disentangle the channels which influence firm 2's decision. See Online Appendix - Section A for a comparison between *in-sight* and *out-of-sight* prices.

[31]The reservation value, $\widehat{v}$, is equal to 1 at $s = 0$ and gradually decreases to $\widehat{v} = p_1$ at $s = \underline{s}$. Since $\widehat{v} > p_1$ in this region, there is no constraint on firm 2's price.

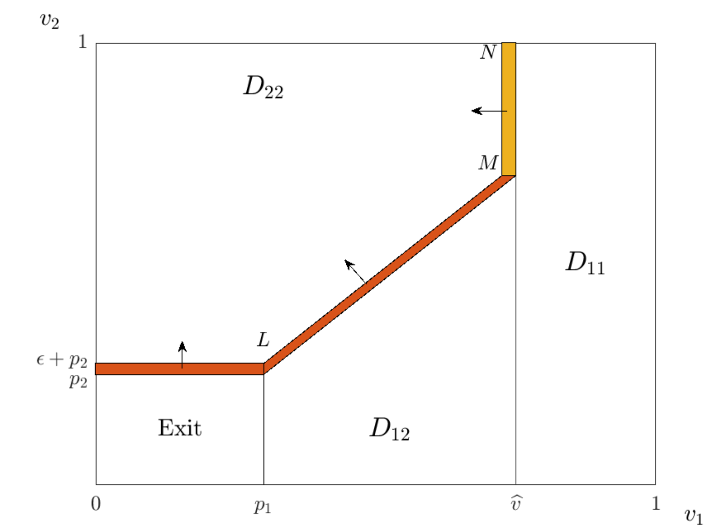


Figure 1: Elasticity of demand: Intuition for prices

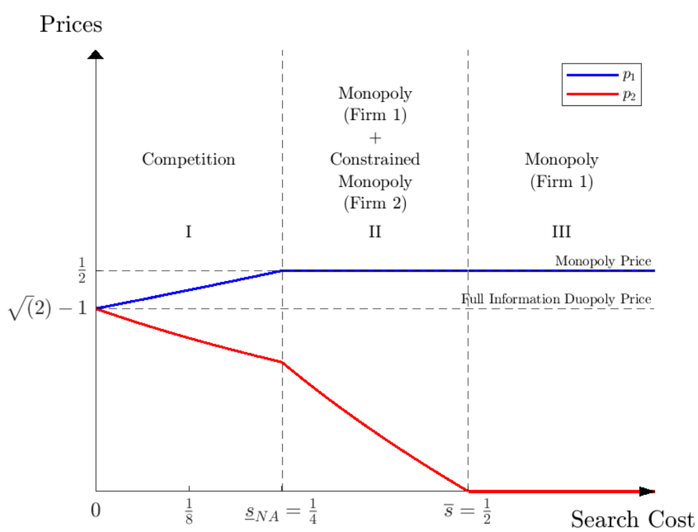


Figure 2: Prices (without auction)

increases. Therefore, even though competition is reducing, we see that p_2 is decreasing with s in equilibrium.[32]

At intermediate search cost ($\underline{s} < s < \overline{s}$), the fraction of "high-interest" visitors hits the limit. In other words, a certain fraction of consumers in the market buy product 1 without considering the

[32] Interestingly, this result is reminiscent of models of switching-cost where firms *subsidise* their price to attract consumers. See, for instance, Valletti (2000), Arie and E Grieco (2014) and Rhodes (2014).

other product while the rest of them find it unaffordable at any price. Therefore, firm 2's demand consists only of consumers who hold in-hand an option of utility zero ($\max\{\widehat{v} - p_1, 0\} = 0$), that is, those who will not return to firm 1 for any offer they get at firm 2. They will either buy product 2 or exit the market. In other words, there is no longer any competition between the two firms. This is as-if there is complete market segmentation as none of the consumers compare the two products while buying. Therefore, both firms would like to set the monopoly price. But since the "Attraction" condition (see Equation 2) binds for $s > \underline{s}$, a high price would not attract any visitors to firm 2. Therefore, firm 2 is forced to choose a constrained-optimal price to attract at least some visitors. Since the revenues would still be positive, firm 2 continues to participate in the market.

At high search costs ($s > \overline{s}$), firm 2 exhausts its ability to 'attract consumers despite search frictions'. The price of firm 2 hits the lower bound of zero. This leaves only one active firm in the market which continues to function as a monopoly.[33]

3.4 Welfare

In this subsection, I analyse the welfare implications in equilibrium. Figure A.2 shows Consumer Surplus (CS_{NA}), Industry Revenue (IR_{NA}) and Total Welfare (TW_{NA}) for $F = U[0, 1]$. As search cost increases, searches are costlier and overall CS is affected negatively. On the other hand, buyers at firm 1 who don't search (D_{11}) avoid this cost. Their numbers increase with rising search cost and hence, contribute positively to CS_{NA}. Since firm 2 has lost its ability to surprise consumers (to extract surplus) due to *in-sight* prices, p_2 falls with search cost, countering the negative effect of search cost on its visitors. In region II, market is segmented and the situation at firm 1 remains constant. However, firm 2 now faces a constraint which forces it to charge a lower price as search cost increases. This favours the consumers substantially until it exits the market in region III. This benefit exactly counteracts the negative impact of search cost for customers of firm 2. Overall, we see a flat curve in region II.

IR_{NA} or Intermediary revenue is maximised at the lowest search cost at which competition is non-existent (market is segmented) and each product caters only to an exclusive group of consumers. TW_{NA} is largely driven by CS_{NA} and reaches its peak at a much lower search cost than the Industry Revenue. This point of maxima for TW_{NA} is also beneficial for firm 2 while firm 1 would be worse-off.

Figure A.2 also plots a measure of positional performance of firms. The *conversion rate* (q) signifies the fraction of visitors that each firm manages to convert into successful sales. This

[33]Note that $s = \frac{1}{2}$ is also the search cost at which market breaks down in Wolinsky (1986) and Armstrong, Vickers, and Zhou (2009).

gives us a measure of efficiency in the matching market.

$$(q_1)_{NA} = D_{11} + D_{12}$$
$$(q_2)_{NA} = \frac{D_{22}}{1 - D_{11}}$$

As search cost increases consumers prefer the option in-hand to searching firm 2 and this drives the conversion rate of firm 1 up. On the other hand, the *information channel* grows stronger with search cost and those who do search probably have a bad offer at firm 1. This can be seen from the increase in firm 2's conversion rate. In region II, firm 2 gradually lowers price to attract consumers which makes its product affordable to more visitors. This culminates with a full conversion rate as $p_2 \to 0$ at $s = \bar{s}$.

A measure of overall market efficiency is the total volume of transactions which is given by the total demand in equilibrium $TR = D_{11} + D_{12} + D_{22}$. This is also equivalent to $1 - p_1 p_2$. The two efficiency measures provide clear model predictions to test empirically. Total transactions reach a maximum at $s = \bar{s}$, driven by the sales at firm 2.

4 Market Equilibrium With Auction

In this section, I introduce a revenue-maximising intermediary that conducts an auction to (i) allocate positions to ads and (ii) determine their commissions.[34]

4.1 Position Auction

Firms' problem. Firms participate in a one-shot auction conducted by intermediary, to procure a position for their ad. Once firms place their bids (their willingness to pay per-click), they learn their positions and commissions. Finally, they set their product prices (see Section 4.4 for a discussion on timing). Formally, firms maximise their profits by choosing their respective auction bids (b_1, b_2) and product prices (p_1, p_2).

$$\max_{p_1, b_1} p_1 \cdot D_1(p_1, p_2, s) - r_1(b_i, b_j, \widehat{b}) \cdot \text{Clicks}_1(p_1, p_2, s) \tag{7}$$

$$\max_{p_2, b_2} p_2 \cdot D_2(p_1, p_2, s) - r_2(b_i, b_j, \widehat{b}) \cdot \text{Clicks}_2(p_1, p_2, s) \tag{8}$$

Intermediary's problem. Intermediary conducts a one-shot Generalised Second Price Auction (GSP) with the objective of maximising its own revenue. She ranks firms based on the expected payment from each firm, that is "Pay Per-Click" (PPC) commission times the number of visitors.

[34] In an intermediate step in Online Appendix - Section C, I characterise the equilibrium product prices in the case of exogenous commissions.

The intermediary is allowed to choose between a monopoly and duopoly product market as well. Ties, in the case of a duopoly, are broken by displaying each possible order with equal probability. The intermediary maximises its revenue by setting the optimal reserve price.

$$\max_{\widehat{b}} \ \pi_m \ = \ r_1(b_i, b_j, \widehat{b}) \cdot \text{Clicks}_1(p_1, p_2, s) \ + \ r_2(b_i, b_j, \widehat{b}) \cdot \text{Clicks}_2(p_1, p_2, s)$$

I assume that the auction reserve price $\widehat{b}$, is announced before the auction, and will be treated as common knowledge. In a GSP auction, firms pay a per-click commission equal to the bid of the next firm in the ordering, that is $r_1 = b_2, \ r_2 = \widehat{b}$.

4.2 Equilibrium

Given consumer demand and firm revenues as an implicit function of prices for each position (see Section 3), I now solve for the optimal bidding behaviour of the two firms.[35] Once I have the optimal bids, I can solve for the optimal auction reserve price.

When firms bid symmetrically,

$$b_i = b_j \ = \ Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big)$$

This is feasible only when

$$\widehat{b} \ \leq \ \frac{Rev_2}{1 - D_{11}}$$

When firms bid asymmetrically,

$$\begin{aligned} b_i \ &\in \ \Big(Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big) \ , \ \infty\Big) \\ b_j \ &\in \ \Big[\widehat{b} \ , \ \ Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big)\Big] \end{aligned}$$

This is feasible only when

$$\widehat{b} \ \leq \ \min\left\{\frac{Rev_1 - Rev_2}{D_{11}}, \frac{Rev_2}{1 - D_{11}}\right\}$$

Table 2 lists the market outcome for different values of the auction reserve price ($\widehat{b}$). The condition for optimal reserve price is derived from firms' participation and incentive compatibility constraints. As the literature has previously shown, I do find that this specification leads to multiple equilibria in pure strategies. When a similar issue of multiplicity arises in the

[35]See Online Appendix - Section B for further discussion of revenue in the absence of the auction.

standard second-price (or Vickrey) auction, the standard equilibrium refinement used focuses on the weakly dominant strategy equilibrium. Analogously, I consider the set of equilibria in undominated strategies. The intuition here is that the bidder plays a strategy such that it will have no regret of losing the top position even if the opponent (other firm) deviates to a slightly lower bid.[36] Similar to the outcome in second-price auction, I find that this refinement rules out asymmetric bidding and we are left with a unique equilibrium outcome in pure strategies, for a given value of search cost (an exogenous parameter).[37] Theorem 2 characterises this outcome. Further, this is the same outcome one obtains when imposing the refinement of 'locally envy-free' strategies (Edelman, Ostrovsky, and Schwarz, 2007). See Section 4.4 for further discussion. I find empirical evidence in section 7 that is consistent with the view that firms' bids are strategic complements and hence, adds support to the choice of equilibrium refinement.

Theorem 2 (Equilibrium: with auction)**.** *In the full equilibrium without auction, optimal bids and auction reserve price in the GSP auction are given by*

$$\begin{aligned} \widehat{b}^* &= \min\left\{Rev_1^* - Rev_2^*, \frac{Rev_2^*}{F(\widehat{v}^*)}\right\} \\ b_i^* &= b_j^* = \left(Rev_1^*, \infty\right) \end{aligned} \tag{9}$$

Firms occupy each position with a probability of one-half. Equilibrium prices in pure strategies for each position are given by

$$\begin{aligned} p_1^* &= \frac{1 - F(\widehat{v}^*) + \int_{p_1^*}^{\widehat{v}^*} F(v_1 - \Delta p)\cdot f(v_1)\cdot dv_1}{f(\widehat{v}^*) - \int_{p_1^*}^{\widehat{v}^*} F(v_1 - \Delta p)\cdot f'(v_1)\cdot dv_1} \\ p_2^* &= \frac{A\cdot[1-F(p^*)]F(p^*) + F(\widehat{v}^*) - F(p_1^*) - \int_{p_1^*}^{\widehat{v}^*} F(v_1 - \Delta p)\cdot f(v_1)\cdot dv_1 + \widehat{b}^* f(\widehat{v}^*)}{A\cdot f(p^*)F(p^*) + f(\widehat{v}^*) - F(p_2^*)f(p_1^*) - \int_{p_1^*}^{\widehat{v}^*} F(v_1 - \Delta p)\cdot f'(v_1)\cdot dv_1} \end{aligned} \tag{10}$$

Equation 10 shows how the asymmetry in commission structure enters the pricing equation. The non-prominent firm faces a marginal cost-like term added to the benchmark expression for prices derived in the absence of the auction (see Theorem 1). This forces firm 2 to raise its price in order to cover this cost. As a result, firm 1 finds some space to increase its own price without the risk of becoming less attractive to consumers. This puts firm 2 at a further competitive disadvantage. To further illustrate the intuitions behind the above result, I again make use of my working example of $F = U[0,1]$.

[36] This requires an assumption that a bidder (firm) exercises caution and doesn't *completely* rule out any action of the other firm. For further discussion, see Brandenburger, Friedenberg, and Keisler (2008) and Hillas and Samet (2020).

[37] Although there might still be multiplicity due to the fact that firm 1's range of bids doesn't have an upper bound, this doesn't play a role in the equilibrium. Nor does it enter into any firms' commission cost.

Table 2: Outcomes for different auction reserve prices

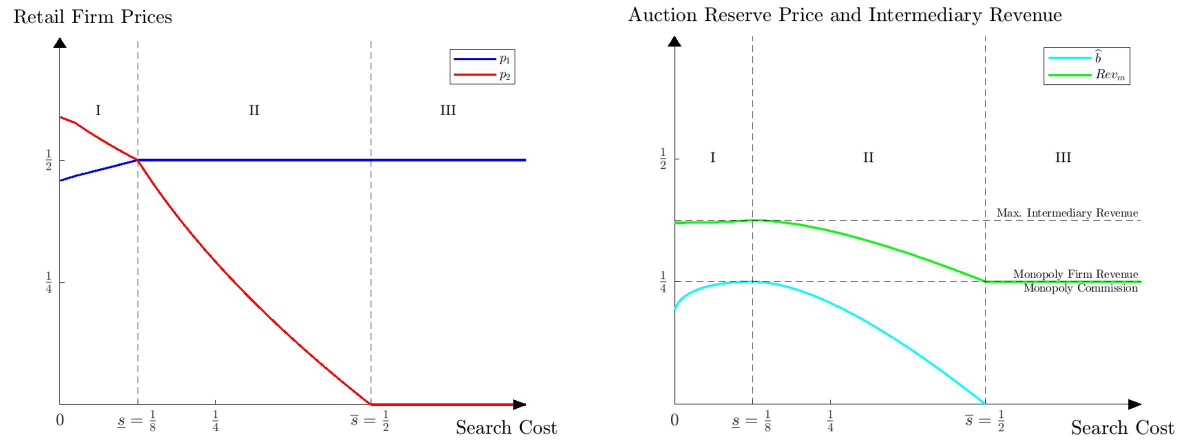
Auction Reserve Price	Firms find ...	Firms' Bids	Firms' Profits
$\widehat{b} > Rev^{Mon}$	neither feasible*	$b_1 = \times$ $b_2 = \times$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = 0$
$\max\left\{\frac{Rev_1 - Rev_2}{D_{11}}, \frac{Rev_2}{1-D_{11}}\right\} < \widehat{b} < Rev^{Mon}$	position 1 feasible	$b_1 = Rev^{Mon}$ $b_2 = Rev^{Mon}$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = Rev^{Mon}$
$\frac{Rev_2}{1-D_{11}} < \widehat{b} < \frac{Rev_1 - Rev_2}{D_{11}}$	position 1 feasible	$b_1 = Rev^{Mon}$ $b_2 = Rev^{Mon}$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = Rev^{Mon}$
$\max\left\{\frac{Rev_2}{2}, \frac{Rev_1 - Rev_2}{D_{11}}\right\} < \widehat{b} < \frac{Rev_2}{1-D_{11}}$	neither feasible	$b_1 = \times$ $b_2 = \times$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = 0$
$\frac{Rev_1 - Rev_2}{D_{11}} < \widehat{b} < \frac{Rev_2}{2} < \frac{Rev_2}{1-D_{11}}$ (this interval has zero measure for $U[0,1]$)	position 2 feasible	$b_1 = \widehat{b}$ $b_2 = \widehat{b}$	$\pi_1 = -\widehat{b}$ $\pi_2 = \min\left\{\frac{1}{2}, 1 - \sqrt{2s}\right\} - \widehat{b}$ $Rev_m = \widehat{b} \leq \frac{Rev^{Mon}}{2}$
$\widehat{b} \in \left[0, \frac{Rev_2}{1-D_{11}}\right]$	both feasible *symmetric bids*	$b_1 = Rev_1 - \pi_2$ $b_2 = Rev_1 - \pi_2$	$\pi_1 = Rev_1 - b_2$ $\pi_2 = Rev_2 - \widehat{b}(1 - D_{11})$ $Rev_m = \frac{b_2 + \widehat{b}(1-D_{11})}{2}$
$\widehat{b} \in \left[0, \min\left\{\frac{Rev_1 - Rev_2}{D_{11}}, \frac{Rev_2}{1-D_{11}}\right\}\right]$	both feasible *asymmetric bids*	$b_1 \in (Rev_1 - \pi_2, \infty)$ $b_2 \in \left[\widehat{b}, Rev_1 - \pi_2\right]$	$\pi_1 = Rev_1 - b_2$ $\pi_2 = Rev_2 - \widehat{b}(1 - D_{11})$ $Rev_m = b_2 + \widehat{b}(1 - D_{11})$

Rev^{Mon}: Revenue of monopoly, Rev_k: Revenue of firm in position k, *feasible: Firm profit ≥ 0

Proposition 2. *When match values are drawn from a Uniform distribution ($v \sim F = U[0,1]$),*

- *the intermediary sets a reserve price to extract all revenue from the non-prominent firm,* $\widehat{b} = \frac{Rev_2}{1-D_{11}}$,
- *firms bids are symmetric and they compete aggressively to end up transferring all the benefits of prominence to the intermediary,* $b_i = b_j = Rev_1$.
- *Hence, firm profits are zero,* $\pi_1 = \pi_2 = 0$,
- *and intermediary revenue equals total firm revenue,* $\pi_m = Rev_1 + Rev_2$

Corollary 2. *Comparison with 'No-Auction' case ($r_1 = r_2 = 0$):*
$p_1 \geq p_{1,NA}$ *and* $p_2 \geq p_{2,NA}$ $\underline{s} = \frac{1-2\widehat{b}^*}{4} < (\underline{s})_{NA}$



(a) Prices

(b) Revenue, commission and profit

Figure 3: Equilibrium: with auction

Figure 3 visualises the results from Proposition 2 and Corollary 2. Since both prices are higher, they hit the monopoly mark at a lower search cost. Further, the reservation value drops with a steeper slope due to relatively larger increase in p_2. Therefore, region I shrinks and market segments for lower value of search cost (see Figure 3).

4.3 Welfare

In this subsection, I analyse the welfare implications of the full equilibrium. Figure A.3 plots Consumer Surplus (CS_{NA}), Industry Revenue (IR_{NA}) and Total Welfare (TW_{NA}) for my working example of $F = U[0, 1]$. The levels of consumer surplus is lower compared to the case of 'No Auction', since the prices are higher in the model with auction. There is a shift in curves to the left as the market segments for a lower search cost ($\underline{s} < \underline{s}_{NA}$).

Empirical Prediction: IR_{NA} or Intermediary revenue is maximised at the lowest search cost at which competition is non-existent (market is segmented) and each product caters only to an exclusive group of consumers. Therefore, if the platform is allowed to choose the optimal search cost (this can be interpreted as the platform choosing its interface), it would choose $\underline{s}$. At this search cost, the model predicts that product prices and conversion rates converge. In other words, conversion rate can be used as a summary statistic to test the mechanism through which ad commission cost passes through into product prices. I find empirical evidence in section 7 that is consistent with the model prediction that when there is no difference in conversion rates across positions, there is no asymmetry in pass-through and hence, for symmetric firms, there should be no difference in their product prices.

4.4 Discussion

4.4.1 Are the equilibria self-fulfilling?

Following the early work in Stahl (1989), treating the first search as free has been a standard assumption in the consumer search literature.[38] Moreover, since consumers encounter online ads free of charge, their search costs are mainly the amount of time spent browsing the product on the Internet. In this context, the assumption of free visit to the prominent firm is motivated by ad listings where websites often host a larger advertisement of a particular firm with product details while the others are shown as smaller thumbnails. Therefore, options may be presented to consumers in an exogenously restricted order, and it costs lesser for consumers to learn the details of the first firm. Another motivating example is that of websites that make some ads less prominent by placing them at a less immediate location, requiring the consumer to scroll down or click on a hyperlink, thus entailing a search cost to visit the second firm. To extend the fit of my model to more general settings, I consider alternative assumptions in this subsubsection using my working example of $F = U[0,1]$.

If $s > 0$ for the prominent as well? Note that consumers still have to follow the displayed order. A positive search cost for firm 1 imposes an additional constraint which determines what fraction of the population prefers to visit firm 1 rather than choose the outside option of zero. Formally, it can be represented as

$$\int_{p_1}^{1} (v_1 - p_1) f(v_1) dv_1 - s > 0$$

This constraint does not bind when $p_1 < 1 - \sqrt{2s}$. For low search cost (region I), the equilibrium prices are the same as in figure 3 as the above constraint does not bind. See Online Appendix - Section D for further discussion on costly first search.

If given a choice, do consumers search in the displayed order? For low search cost (region I), p_2 is greater than p_1 and consumers prefer to follow the order in which firms are displayed.

At $s = \underline{s}$, firms charge an identical price, $p_1 = p_2$. Hence, consumers are indifferent between starting from either firm and if consumers can choose which firm to visit, we will have random search. Thus, firms' prices will follow Wolinsky (1986) and have a value of $1 - \sqrt{2s}$ for $\underline{s} < s < \bar{s}$. At $s = \underline{s} = \frac{1}{8}$, this value coincides with the monopoly price, $\frac{1}{2}$.

[38] Recent work on consumer search has often assumed first search as free. See, for instance, Janssen, Moraga-González, and Wildenbeest (2007), Janssen and Parakhonyak (2013), Ding and Zhang (2018), Janssen and Shelegia (2020).

4.4.2 Superstar and Fringe

In my model, the intermediary would prefer to design the market at a higher search cost than the welfare-maximising level. Also, note that firms' revenues get more dispersed with search cost. Another factor that affects revenue dispersion is the per-click commission cost. This asymmetry in cost structure significantly changes firms' price setting problem, as captured starkly in the duopoly model. Potentially, a superstar firm may have sufficient funds to *enter* the top positions by paying the *fixed cost*. This pushes the fringe firms to lower positions. Since the profits are distributed in order of position, this result highlights a mechanism through which a profit-maximising intermediary may exacerbate the gap between superstar and fringe firms and provide a rationale for growing market concentration. See Online Appendix - Section E and F for a discussion the timing of the game and auction format respectively.

5 Asymmetric Firms

Firms that compete in the position auctions may be vertically differentiated as well. In this section, I relax the symmetric nature of the firms.I differentiate firms by the maximum value that any consumer can realise at a firm. I refer to this dimension as the *quality* of a firm (or product).

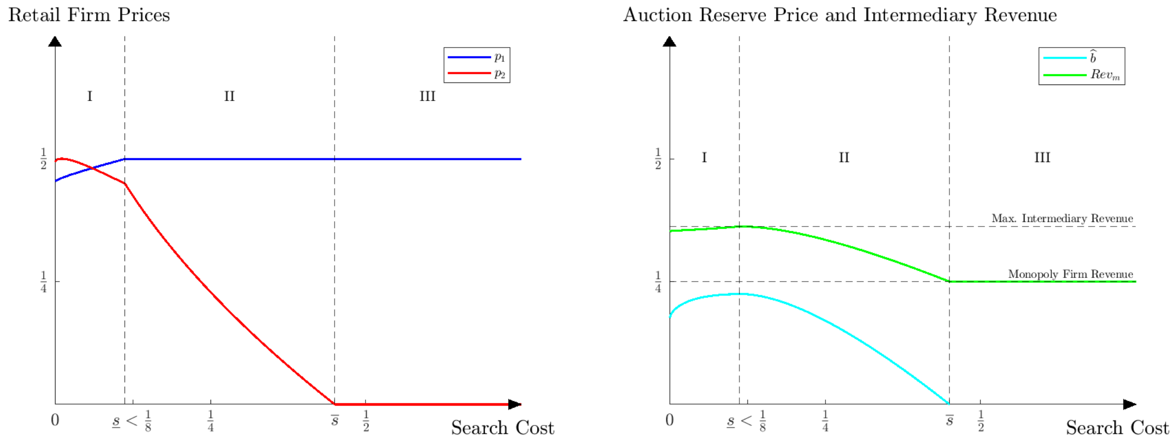
Theorem 3 characterises the equilibrium for asymmetric firms. Table A.1 shows the market outcome for different values of the auction reserve price ($\widehat{b}$). Without loss of generality, let firm i have the ability to generate higher revenue than firm j under the same conditions (of position and auction reserve price). The reservation value for the search rule is now given by

$$\widetilde{v}_\alpha \ = \ \alpha + \Delta p - \sqrt{2\alpha s}$$

Proposition 3. *There exists no symmetric equilibrium. Asymmetric equilibrium in undominated pure-strategies for firms i, j such that firm i is more relevant or has higher quality is as follows. Optimal bids and auction reserve price in the GSP auction are given by*

$$\begin{aligned} \widehat{b} \ &= \ \frac{Rev_{2,j}}{1 - D_{11,i}} \\ b_i \ &= \ (Rev_{1,j} - \pi_{2,j}, \ \infty) \\ b_j \ &= \ Rev_{1,i} - \pi_{2,i} \end{aligned} \tag{11}$$

The intuition for the non-existence of symmetric equilibrium is a result of the additional asymmetry in firms' abilities on top of the positional asymmetry. The positional heterogeneity



(a) Prices

(b) Commission

Figure 4: Quality: With auction

and quality are in resonance and firm i procures the first position by bidding higher and earning higher in equilibrium.

In Figure 4, I illustrate the the equilibrium outcomes, when firms offer products of heterogeneous quality. I assume that one of the products is on average of higher quality by assuming that consumers draw their valuations from a distributions $v_1 \sim U[0,1]$ and $v_2 \sim U[0,\alpha]$ where $\alpha \in \mathbb{R}_+$.

In Online Appendix - Section G, I consider a second dimension, where I differente firms by the maximum measure of customers that they can attract, for any price. I call this the *relevance* of a firm, capturing horizontal differentiation. In both the exercises, I show that the equilibrium outcomes are similar whether firms differ by *relevance* or *quality*. The main and common takeaway is that there is now much less of threat to the prominent firm from firm 2 because firm 2 is less appealing to consumers due to lower *relevance* or *quality*. This amplifies the competitive disadvantage for firm 2. Further, the intermediary prefers a monopoly to a duopoly at a lower search cost than in the symmetric case ($\overline{s} < \frac{1}{2}$), as the ability of firm 2 to generate revenue exhausts for a lower s due to the amplification of asymmetries.

6 Platform Revenue Models and Welfare

In my baseline model, I studied the *pay per-click* advertisement revenue model implemented by the intermediary, where each firm bids their willingness to pay in a Generalised Second Price (GSP) auction, and eventually pays a commission for each click it receives. Solving this model, I showed that the consumer surplus is negatively affected due to the asymmetric pass-through of ad commission and the resulting rise in product prices. In this section, I consider variants of the intermediary's revenue model, while still allowing platforms to rank firms, which might relax the asymmetry in ad commission pass-through. First, I study the (i) *pay per-sale* and (ii) *consumer subscription fee* models, in place of the *pay per-click* model.[39] Second, I study a combined-revenue model of *pay per-click* and *consumer subscription fee*.Third, in Online Appendix - Section H, I also study the Generalised First Price auction, in place of the GSP (leaving the *pay per-click* design unchanged).

6.1 Alternative Revenue Models

6.1.1 Per-Sale Commission

In the following exercise, I analyse the implications on welfare when the intermediary implements the *pay per-sale* revenue model *instead* of the *pay per-click* model. Unlike the per-click model where the intermediary was interested in the number of clicks that firm 2 receives, the intermediary is now interested in the number of purchases made at firm 2. Since the ad cost of the non-prominent firm is directly determined by the intermediary's auction reserve price, the monopoly intermediary sets $\widehat{b}$ equal to the anticipated p_2, thus extracting all revenue from firm 2. In turn, this increase in "marginal cost" raises p_2 to the limit where firm 2 can just attract some positive measure of consumers, conditional on the intermediary preferring to operate a duopoly instead of a monopoly. This limit is reached when p_2 equals $\widehat{v}$. This strongly suppresses firm 2's ability to compete. Therefore, the prominent firm is at further competitive advantage now compared to the pay per-click model.[40]

Proposition 4. *In equilibrium, $\widehat{b} = p_2 = \widehat{v}$. There is search in equilibrium only by consumers who cannot afford product 1. So, market is segmented for any $s > 0$ (i.e. $\underline{s} = 0$) and firm 1 charges the monopoly price.*

Corollary 3. *The pay-per-sale (pps) commission structure makes the firm outcomes more unequal than the pay-per-click (ppc) commission structure. Thus, the prominent firm focuses its sales on fewer, but*

[39] A random display-structure of ads is less appealing for two reasons: (i) it eliminates one of the main benefits of the intermediary, which is to use the data collected to recommend more suitable ads, and (ii) while an intermediary may claim to randomise its listings, it may continue to do some selection and this is hard to monitor for firms and policymakers.

[40] Interestingly, this result is reminiscent of Petrikaitė (2018), where a multi-product monopoly sets the search cost just high enough to induce market segmentation. This market resemblance to a multi-product monopolist provides an alternative intuition for the fall in competition under the pay per-sale commission structure.

more valuable, consumers and extracts more rent from them.

$$\begin{aligned} p_1^{pps} &\geq p_1^{ppc} \\ p_2^{pps} &\geq p_2^{ppc} \\ Rev_1^{pps} &\geq Rev_1^{ppc} \\ Rev_2^{pps} &\leq Rev_2^{ppc} \end{aligned}$$

In terms of welfare, on the one hand, the *pps* commission structure incentivises the intermediary to increase sales and not search, which reduces the consumer-surplus loss due to excessive search, compared to the *ppc* structure. On the other hand, less searchers means that it also reduces the total number of transactions and concentrates more market power with the prominent firm, thus making the market less competitive and decreasing the total industry revenue and consumer surplus. Except for very low search costs, the first force dominates and consumers are better-off, while the inequality between firms widens. Figure A.5 illustrates this trade-off in welfare considerations that a regulator might face, for my working example of $F = U[0, 1]$.

6.1.2 Consumer Subscription

In the following exercise, I analyse the implications on welfare when the intermediary implements the *consumer subscription fee* revenue model, *instead* of the *pay per-click* model. For this exercise, I assume that the subscription fee is paid prior to making any purchase. Hence, consumers internalise this fee while making their decisions. Formally, consumers draw their product value from the distribution

$$v_1, v_2 \sim U[0, 1 - sub]$$

where sub denotes the homogeneous subscription fee that the consumers pay *ex-ante* (prior to making a purchase). Figure A.6 plots CS and IR for $F = U[0, 1]$, at $s = \frac{1}{8}$, for different values of subscription fee.

Product prices decrease with rise in the *ex-ante* subscription fee, since consumers' maximum willingness to pay falls. This, combined with fewer searchers (as in the *pps* model), implies that CS increases with higher subscription fee (see Figure A.6). However, due to the fall in total transactions, it implies a fall in industry revenue. Hence, this alternative revenue model could also be consumer-surplus improving, at the expense of the intermediary's surplus.

Figure 5 summarises and compares the different platform revenue models discussed above, for my working example of $F = U[0, 1]$ at $\underline{s} = \frac{1}{8}$. As we move down the panels in Figure 5, we see a higher share of the total surplus with the consumers, at the expense of the intermediary. From

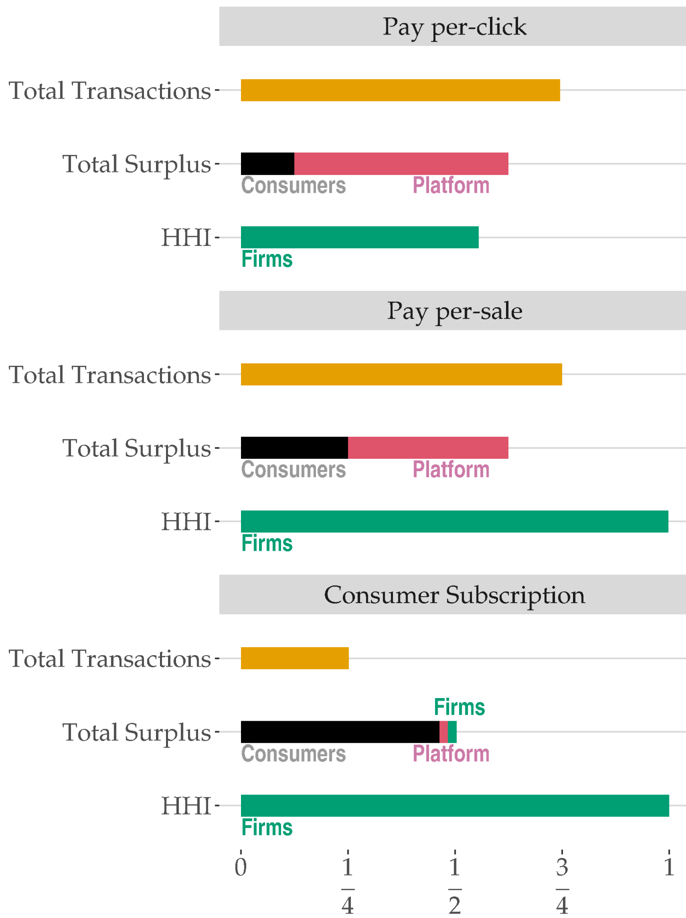


Figure 5: Static vs Dynamic considerations: Surplus and Market Concentration

a policy perspective, this suggests that a policymaker should prefer to implement the alternative revenue models. However, note that as we move down the panels, we also see a fall in total transactions and a rise in the HHI concentration index of the product market. Contrary to the previous insight, this suggests to the policymaker that market competition will suffer in the long-run in the alternative revenue models. Therefore, the weight placed on different welfare-criteria and time horizons (static and dynamic considerations) will be crucial in determining the socially-optimal mechanism.

6.2 Combining Platform Revenue Models

In the following exercise, I analyse an intermediary with two revenue sources: one, where it conducts a GSP to determine advertisement commissions from firms, and two, it obtains a share of the surplus from consumers through a subscription fee prior to purchase.

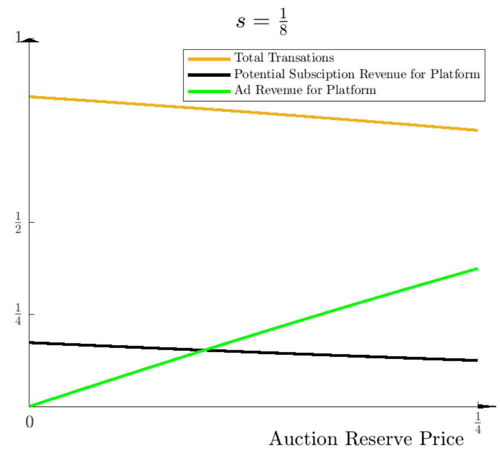


Figure 6: Platform prefers Ad revenue more than Consumer Subscription

The trade-off here for the intermediary is that while it can potentially gain more revenue from consumers when they are left with higher surplus (through a lower subscription fee), it has to enable and enlarge this new source by diminishing its old source of ad commissions. This can happen through two channels: consumers who do not buy (extensive margin), do not have any surplus that they can share with the intermediary. Lowering its commissions on the firm-side reduces prices, leaving more surplus with the consumers (intensive margin), which can be extracted by the intermediary.

Due to the above trade-off, the intermediary is particularly interested in increasing the purchases at the non-prominent position. This is because consumers of the non-prominent product are left with a higher surplus, since its price is lower due to the asymmetric effect of the Sponsored Ad auction (see Section 4). As a result, the intermediary reduces $\widehat{b}$, which increases the profits at non-prominent position. This reduces the incentive for a firm to be prominent, which also reduces the commissions chargeable (or Incentive Compatible) at the prominent position. Thus, it leads to a fall in revenue from advertising commissions.

Figure 6 shows that the rate of increase in surplus with consumers, due to lower ad commissions, is lower than the rate of decrease in ad revenue for the intermediary, in my working example of $F = U[0,1]$ (this result is robust to different values of search cost). Therefore, a revenue-maximising intermediary would prefer to focus its revenue generation strategy solely on the ad commissions, even when it is allowed to appropriate a share of consumer surplus. This result provides additional rationale for the cross-subsidisation behaviour of platforms in two-sided markets.[41] Note that in a competitive environment with multiple platforms, a platform

[41]See, for instance, Baye and Morgan (2001), Caillaud and Jullien (2003), and Chen and Rey (2019) for some previous

would be more inclined to forgo the *consumer subscription fee* as a source of revenue. However, the effect is ambiguous and is beyond the scope of this paper.

7 Data and Evidence

In this section, I use clickstream data from the US to test the hypotheses developed in Section 4. The model posits that in the platform-optimal design (at $s = \underline{s}$), compared to a similar ad in a non-prominent position, an ad in the prominent position (i) pays higher commission per-click, (ii) receives more clicks, (iii) converts a similar fraction of clickers, (iv) charges a similar price, and (v) receives the first click from a user session. To evaluate the effect of prominence on the above outcomes, I employ the strategy of selection on observables. The rest of this section proceeds as follows: First, I discuss the strengths and limitations of the dataset. Second, I summarise and describe the patterns in the data. Finally, I test the above hypotheses.

7.1 Details and Background

The dataset (Diemert Eustache, Meynet Julien, Galland, and Lefortier, 2017) contains 30 days of user traffic and the ads shown to them on an anonymous platform in the US. It provides information on both ad commission and user behaviour, which provides a unique opportunity to find evidence on the interaction between consumers and firms in the sponsored ad setting.[42] A typical user who opens the platform (i) sees ordered ads, (ii) might click to search and learn details about the product and (iii) might purchase. Each observation corresponds to an ad that was displayed to a user (also referred to as an *impression*). Data has been sub-sampled and anonymized so as to not disclose proprietary elements.[43] We observe a unique identifier for each user, a unique identifier for each ad campaign,[44] whether the ad was clicked, purchased, time of click, time of purchase, position at which the ad was shown, position of the click in consumers' search order, whether eventual purchase is attributed to the platform, the commission paid to the intermediary for that click and the cost-per-order borne by the firm.

Before advancing to the empirical analysis, highlighting two caveats of the data are in order. Firstly, the data doesn't clarify the auction format used. Since ads are largely auctioned off on

work on cross-subsidisation.

[42] To my knowledge, this data has not been used for economic or marketing analysis. The dataset was originally compiled by a large Digital Marketing agency to train and develop efficient Machine Learning models to evaluate its performance and improve its design. See Appendix B for more details.

[43] Note that the organisation has not formally revealed if the sampling was random.

[44] The word "campaign" is commonly used in the digital marketing/e-commerce industry to refer to ads. For e.g., registered users who want to serve Google Ads can create their own advertisement *campaigns* (source: https://support.google.com/google-ads/, accessed: May 18, 2021). Note that this identifier could potentially refer to retailer × product. However, it is unlikely to be the case that a firm uses two different campaigns for exactly the same product on the same platform, within a span of 30 days. Therefore, for my analysis, I assume that each product is produced by a single-product firm.

Table 3: Data Description

	#
Users	985,273
Users who made at least one click	480,536
Users who made at least one purchase	42,673
# of ads	675
Total observations	1,612,594

a per-click second-price design, I assume that this is the case.[45] Following my model, I also abstract from any dynamic strategic considerations that firms may have in their bidding process. Secondly, a caveat in most datasets of online advertising with position auction, as in mine, is that they do not reveal the posted price of products. However, this dataset provides some additional information which I use to construct a notion of product prices. I use the *cost per order* (CPO) borne by the firm as a proxy for product price. In standard industry accounting practice, CPO includes customer acquisition costs, packaging costs, fulfilment costs, shipping costs, COGS and storage costs, averaged per-order. The following analyses are subject to these assumptions.

7.2 Descriptive Statistics

I restrict the dataset to all observations where exactly two ads were listed. Even though this removes useful information, it allows me to identify competitors cleanly and to match the empirical setting with that of the theoretical model. I also drop observations (few in number) where more than one purchase was made, to avoid concerns of ads displaying complementary products. Note that sometimes a user may click on an ad show on the platform but buy the product later from a different source. To avoid concerns of showrooming, I restrict my attention to purchases that were attributed to the platform (this is a variable that the marketing agency provides). Table 3 presents a summary of the dataset. Note that the data is essentially a cross-section of users encountering a panel of ads across time.

Table 4 presents a summary of the click and purchase behaviour of consumers. Around half of the platform-users do not click on the ads displayed, consistent with the notion that display ads are used to target consumers at early stages of the purchase process and have a lower click-through rate (CMA, 2020a). It also shows that consumers' order of clicking on ads always follows the order in which the ads were displayed. Figure 7 provides a description of the main empirical variables, by comparing an ad that is displayed in a prominent position with the same ad when

[45]See, for instance, https://support.google.com/adsense/answer/160525?hl=en.

Table 4: Prominence & User Behaviour

Activity		# users	users, in %
Click			
see both ads		985,273	100.00
visit none		508,294	51.59
visit only first ad		368,186	37.37
visit second ad,	but not first	0	0.00
visit both ads		108,793	11.04
Conversion			
visit only first ad,	buy from first ad	12,109	2.61
visit both ads,	buy from first ad	6,888	1.49
visit both ads,	buy from second ad	4,921	1.06
visit both ads,	buy from none	439,899	94.84
visit none,	buy from any one	0	0.00

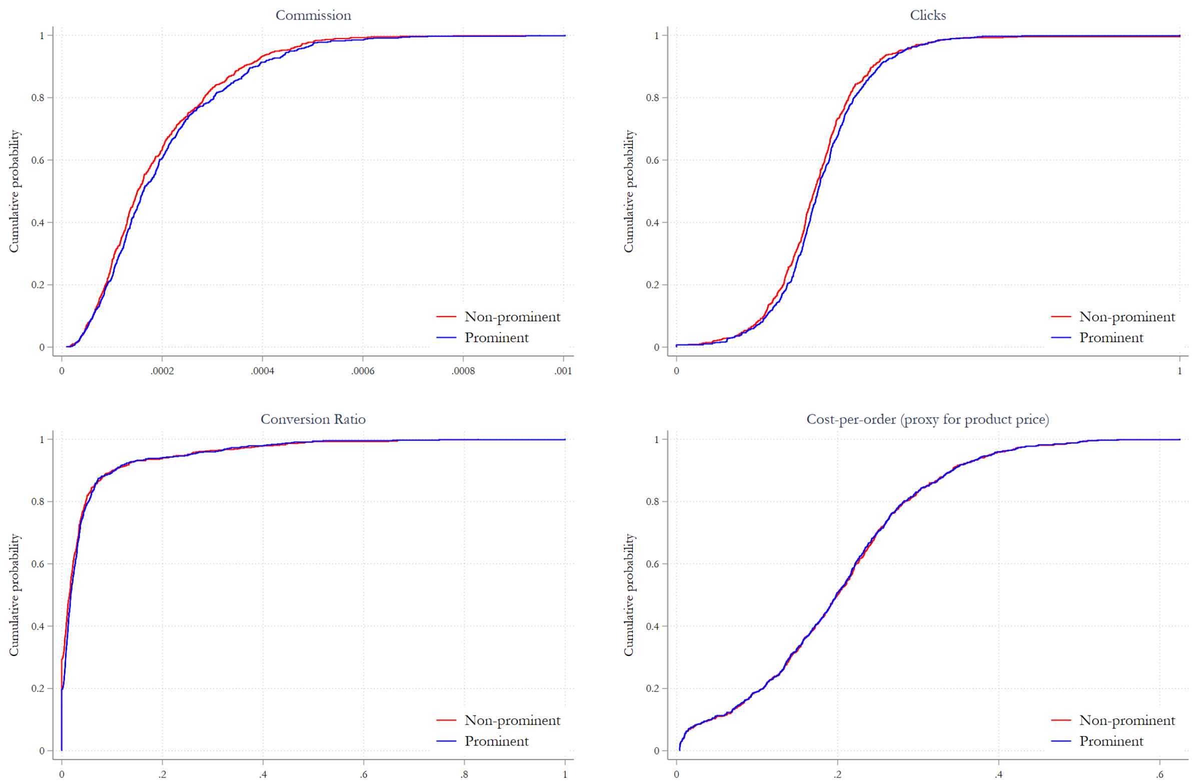
Notes. Conversion: purchases made, conditional on clicking on ad. The third column in the bottom panel on Conversion of clickers into buyers reports percentage of purchases among users who made at least one click.

it is displayed in a non-prominent position.

I follow the notation from Section 2 for users' purchase behaviour. D_{11} denotes users who visited only the firm in position 1 and bought from it. D_{12} denotes users who visited both firms but eventually bought from firm 1. D_{22} denotes users who visited both firms and eventually bought from firm 2. D_0 denotes users who visited a firm but didn't purchase from any. Table 4 presents the distribution of users across these categories. It shows that there are more purchases of the product of the ad in prominent position: $D_{11} > D_{12} > D_{22}$. It also shows that a large fraction of users do not click or do not buy after clicking, consistent with the intuition for display advertisements.

Figures 8 and 9 show the patterns of search and purchase activity across all products at the daily and hourly level. The day-level plots show cyclical patterns with a frequency of approximately seven days. This is consistent with the phenomenon of higher activity during weekends. At the hourly level, we see a large difference between daytime and night-time activity, with the peak occurring around 8 pm, consistent with the notion of prime-time. Hence, to address concerns of time-specific factors affecting market outcomes, I will control for these patterns in my empirical strategy.

Figure 7: CDF plots of the main empirical variables, for the same ad in prominent and non-prominent position



Notes. Cumulative probability distribution of variables plotted after controlling for ad fixed effects: (i) commission paid by firm to platform, (ii) clicks received, (iii) fraction of clickers converted into buyers, and (iv) cost-per-order, which represents the total non-production cost (e.g., shipping, storage, etc.) made and is used as a proxy for price. The data provider has normalised variables (i) and (iv) to be between 0 and 1.

7.3 Empirical Analysis

To analyse the effect of prominence on market outcomes, I estimate the following equation

$$y_{ijt} \;=\; \alpha_j + \alpha_t + \beta \cdot prominence_{ijt} + \epsilon_{ijt} \tag{12}$$

for user i facing ad j at time t. The outcome variables are denoted by y and include (i) commission per-click, (ii) a dummy =1 if the ad received a click, (iii) a dummy = 1 if the ad converted the individual, conditional on receiving a click (conversion rate), (iv) cost-per-order (proxy for price), and (v) a dummy =1 if the ad received the first click by a user in their search session. The explanatory variable *prominence* is a dummy =1 if ad is displayed in the prominent position. The coefficient of interest is β. Due to the lack of an experimental randomisation, I interpret the estimate as non-causal, but informative about the differences between similar ads which differ only by their display position (prominence). I include fixed effects to control for unobservables:

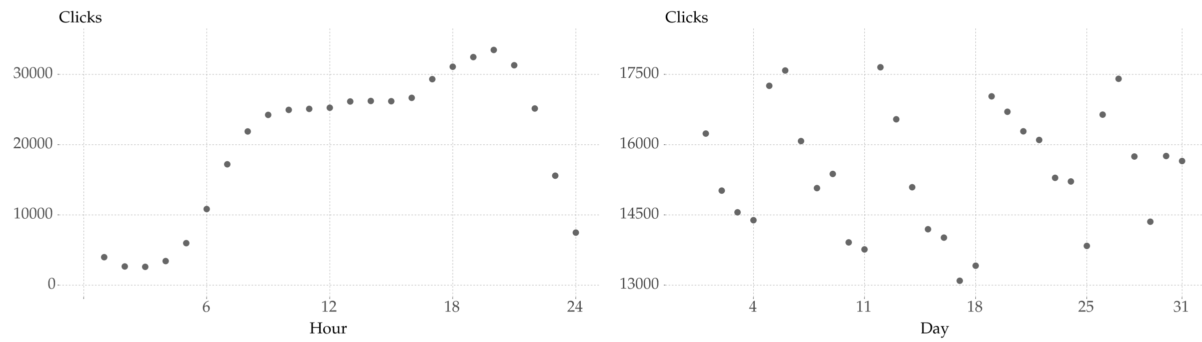


Figure 8: Click behaviour

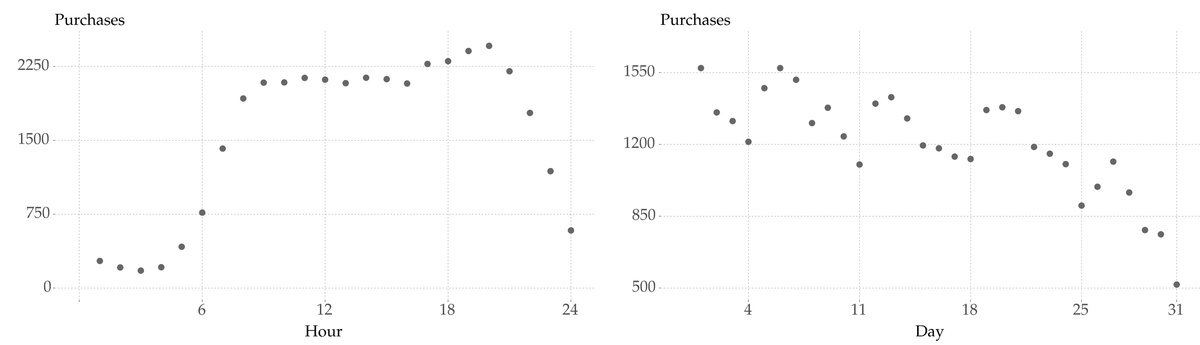


Figure 9: Purchase behaviour

α_j are ad FE to account for unobserved ad characteristics such as ad design, product quality, industry/sector, and α_t are day of the week FE and hour of the day FE to account for the seasonality highlighted in the previous subsection. Standard errors are clustered at the ad-level.

Table 5 presents results from the estimation of Equation 12. Each column corresponds to each one of the model hypotheses highlighted above. The table suggests that the observations in the data are consistent with the model at the search cost where Intermediary revenue is maximised ($s = \underline{s}$). Specifically, compared to a similar ad in non-prominent position, the ad in prominent position (i) pays higher commission, (ii) receives more clicks, (iii) converts a similar fraction of clickers, (iv) charges a similar (proxy for) price, and (v) receives the first click of a user.

As the mechanism of asymmetric pass-through of commissions is independent of the feature of price visibility, the qualitative nature of the estimates in Table 5 should be robust to any considerations regarding price visibility on the anonymous platform. Further, as I find lot of variation in the proxy variable for product price, both with position and commission, even when I look at the same ad repeatedly over the month, any concern about firms' commitment to prices

Table 5: Effect of prominence

	(1)	(2)	(3)	(4)	(5)
Outcome:	commission	clicks	conversion rate	cost-per-order	first click
prominence	0.0746***	0.00808***	-0.00130	-0.0389	1***
	(0.021)	(0.002)	(0.001)	(0.026)	(0.000)
Observations	1,594,107	1,594,107	460,785	1,594,107	460,785
R-squared	0.050	0.029	0.142	0.823	1.000
mean(y)	1.921	0.297	0.0781	20.80	
sd(y)	5.395	0.457	0.268	12.07	
ad FE	✓	✓	✓	✓	✓
day of the week FE	✓	✓	✓	✓	✓
hour of the day FE	✓	✓	✓	✓	✓

as the driver of the null result in Column (4) in Table 5 is unlikely. However, in general, one should be careful to not interpret my empirical evidence as conclusive; this topic deserves further analysis.

Heterogeneity. As additional evidence, I hypothesise that prominence is more valuable when the demography is more likely to engage (in the form of higher clicks or purchases). For example, users visiting a website on "tips to purchase a wi-fi modem" are more valuable for a wi-fi service provider than users visiting other websites, as they are more likely to click and purchase from an ad on wi-fi. Conditional on observing higher total engagement over the two positions, Table B.1 confirms that the prominent firms pay a higher commission compared to the case when they appear with ad-pairs that receive low user engagement.

Evidence on bidders' strategic interactions and equilibrium refinement. De Paula and Tang (2012) show that, under certain conditions, the second-order moments of equilibrium outcomes (commissions, in my setting) can be informative about the strategic interactions in the actions chosen (bids, in my setting) and therefore, about the presence of multiple equilibria in outcomes (commissions). More precisely, the sign of the moments can be informative about whether bids are strategic complements or substitutes. The key assumption required is that consumers' match values are independent across ads within a market, a standard assumption in consumer search frameworks that is also used in my theoretical model (see section 2).

Relying on this assumption, Table B.2 shows that firms' bids are strategic complements (in other words, there is a positive correlation between the bids of the firm that ends ups being prominent and the firm that ends up being non-prominent), which provides support to the equilibrium refinement used in Section 4: of finding firms' bidding equilibrium in undominated strategies. To address concerns that this result might be driven by some highly valuable auctions, I carry out the same exercise by focusing on a time period (the last week) with low user engagement (see Figure 9). Table B.3 shows the baseline results hold even when user engagement is relatively lower, thus adding support to the interpretation that bidding strategies are, indeed, strategic complements.

8 Conclusion and Discussion

The two-sided nature of online marketplaces makes the intermediary a key player in influencing economic outcomes. This paper presents a model of consumer-firm and firm-intermediary interaction to illustrate the tradeoffs for each player in this setting. I consider an ordered search model where consumers discover independent valuations of each product by paying the search cost while the intermediary lists the products by conducting an auction. Product prices being observable prior to search plays an important role in determining consumer demand as well as profits of firms and the intermediary. Moreover, the order of product prices in the list are determined by the search cost and auction reserve price. Since the ad commission takes up the form of fixed cost for the first firm and marginal cost for the second, this changes the pricing equation for each and in turn, the demand and profits. Hence, even when there is no friction in access to product prices, and even when the intermediary is not directly competing with firms, there can be an exacerbation of market concentration.

I show that adopting a *pay-per sale* ad sale model by the platform can improve consumer surplus, but this comes at the expense of the intermediary. My results suggest that another policy-candidate to improve consumer welfare is to monitor price visibility. This is promising, on the one hand, because even when consumers are forced to search (if their realisation at firm 1 is low), *in-sight* prices force firm 2 to lower its price and prevent surplus extraction from the held-up consumers. Further, this is straightforward to implement in practice by holding platforms liable to minor design changes. However, it warrants caution, since, on the other hand, *in-sight* prices reduce the number of searches or product exploration, and in the long run, it might induce entry barriers. Therefore, evidence on long-run outcomes is required to fully understand its welfare implications.

My model gives predictions for two measures, the conversion rate of firms and the total volume of transactions. This can provide a benchmark for the empirical estimation of the impact of prominence and ad commissions. In practice, the *conversion rate* (ratio of buyers to visitors) could be a useful metric for managers and policymakers as well. Essentially, higher *conversion rate* is what matters to firms, when evaluating their investment in advertising. To evaluate ad commissions' impact on pricing, a firm's conversion rate provides the intuition for the pass-through from ad commissions (paid per-click) to prices (charged per-sale). In another perspective, the *conversion rate* for firm 2 is important for welfare as ad commission plays the role of a marginal cost. However, burdened by the commission cost, firm 2 is forced to raise p_2 which lowers both its conversion rate and the overall transactions in the market, thereby exacerbating the dispersion in firms' profits. Further, this rise in p_2 gives room for firm 1 to raise p_1 without losing consumers, thus increasing its market power. This asymmetric nature of the market provides a novel rationale for growing market concentration, and can be captured by the *conversion rate*.

Sponsored ads are largely perceived as playing an informative role for consumers, in guiding them towards products that they are seeking. However, recent discussions (see, for instance, Heidhues and Kőszegi, 2018) highlight the role of the intermediary in steering consumers toward ads that are intermediary-preferred, thereby persuading or nudging consumers to buy a higher-priced product. In my paper, I show that the intermediary-preferred product could endogenously obtain a prominent position and set a lower price, thereby persuading rational consumers (without nudging) to buy the prominent product and generate higher platform and firm revenues.

Some of the insights gained from this model may also apply to other situations, where sellers pay for their products to be displayed in a prominent position. For example, in offline supermarkets, firms advertise at the entrance or near the cash counter. Further, the store may order these ads in different positions based on the flow of traffic within a store and the commission promised by each manufacturer. Some other examples include publishers paying book-stores for promotion, more prominent ads being more expensive in brochures/menus, and eBay offering sellers the option to list their products prominently for an extra fee. With suitable adaptation, my framework can be applied in a related set of circumstances wherein market participants or public authorities seek to influence consumer choice by framing the order in which options are presented. For instance, the presentation of options for savings plans, healthy eating, advertising and its regulation, or the operation of commission schemes for *e-commerce* platforms.[46]

The topic of digital marketplaces deserves further research theoretically and empirically. Making a firm prominent will have an impact on firm entry. Since the top firm has to pay a fixed cost and, at high search costs, can enjoy a large market share, one may be concerned with the potential for entry deterrence which may affect product variety negatively.[47] On the other hand, this might increase efficiency because free entry may result in excess entry, for instance, in the random-search case (Anderson and Renault, 1999). Empirically estimating the pass-through of ad commissions remains elusive and could provide insights into the strength of each mechanism. Further, we need welfare analyses which take into account the distribution of outcomes over heterogeneous agents. For instance, in my model, I find that the gap between

[46]See, for instance, Thaler and Sunstein (2009) for an overview of some of these issues.
[47]See, for instance, Lee and Musolff (2021) for some recent related work.

firms' revenues widens with search cost. This result appeals to recent discussions on increasing market concentration and implores further research. Studying a dynamic model to highlight the effect of endogenous consumer loyalty can also help understand the long-run implications and can provide additional insights on firm strategy in the initial periods.

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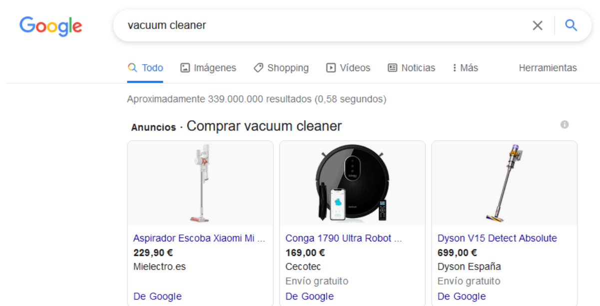
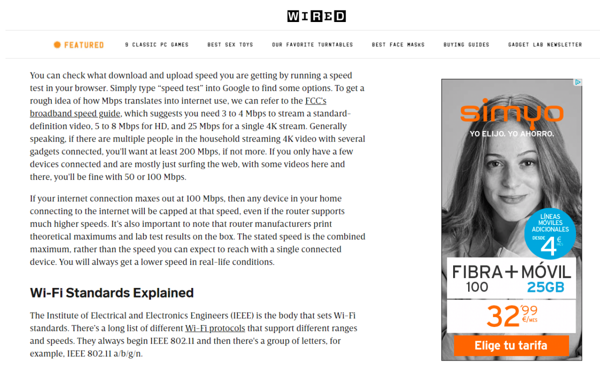
A Appendix: Theoretical Analysis

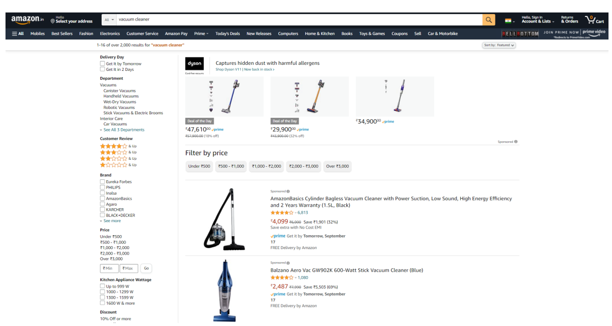
A.1 Figures and Tables



Source: https://www.google.com/ (Accessed: 16 Sep, 2021)

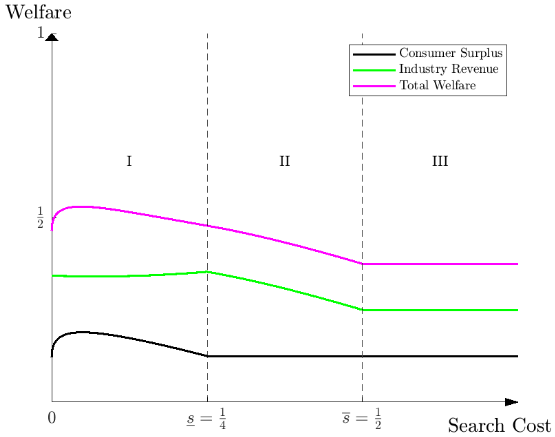


Source: https://www.wired.com/story/how-to-buy-a-router/ (Accessed: 16 Sep, 2021)

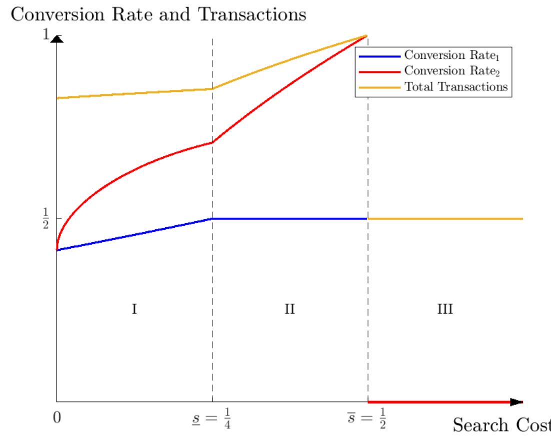


Source: https://www.amazon.in/ (Accessed: 16 Sep, 2021)

Figure A.1: A snapshot of sponsored results: Prices are often observable costlessly on digital platforms

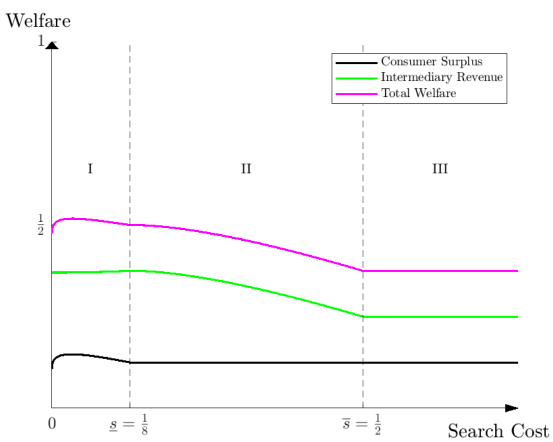


(a) Welfare

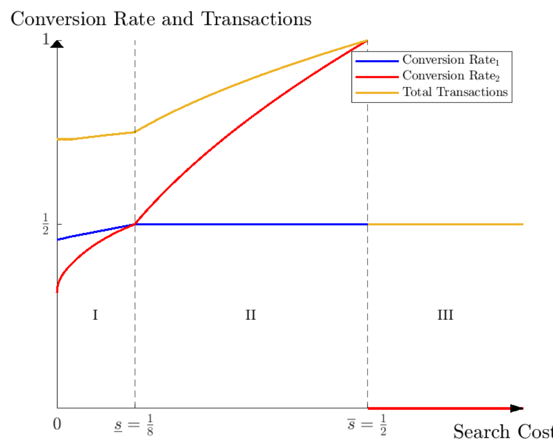


(b) Conversion Rate

Figure A.2: Welfare and Efficiency: 'No Auction'



(a) Welfare



(b) Conversion Rate

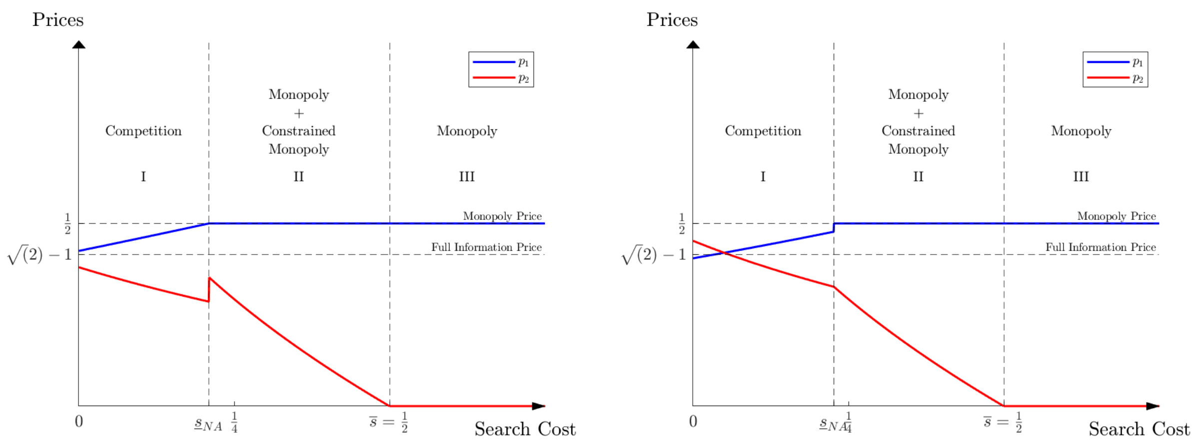
Figure A.3: Welfare and Efficiency: With Auction

Table A.1: Outcomes for different auction reserve prices: Asymmetric firms

Auction Reserve Price	Firms find ...	Firms' Bids	Firms' Profits
$\widehat{b} > Rev_{1,j}$	neither feasible*	$b_1 = \times$ $b_2 = \times$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = 0$
$\max\left\{\frac{Rev_{1,i}-Rev_{2,i}}{D_{11,j}}, \frac{Rev_{2,j}}{1-D_{11,i}}\right\} < \widehat{b} < Rev_{1,j}$	position 1 feasible	$b_1 = \frac{Rev^{Mon}}{2}$ $b_2 = \frac{Rev^{Mon}}{2}$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = \frac{Rev^{Mon}}{2}$
$\frac{Rev_{2,j}}{1-D_{11,i}} < \widehat{b} < \frac{Rev_{1,i}-Rev_{2,i}}{D_{11,j}}$	position 1 feasible	$b_1 = \frac{Rev^{Mon}}{2}$ $b_2 = \frac{Rev^{Mon}}{2}$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = \frac{Rev^{Mon}}{2}$
$\widehat{b} \in \left[0, \min\left\{\frac{Rev_{1,i}-Rev_{2,i}}{D_{11,j}}, \frac{Rev_{2,j}}{1-D_{11,i}}\right\}\right]$	both feasible asymmetric bids	$b_1 \in (Rev_{1,j} - \pi_{2,j}, \infty)$ $b_2 \in \left[\widehat{b}, Rev_{1,i} - \pi_{2,i}\right]$	$\pi_1 = Rev_{1,i} - b_2$ $\pi_2 = Rev_{2,j} - \widehat{b}(1 - D_{11,i})$ $Rev_m = b_2 + \widehat{b}(1 - D_{11,i})$

Rev^{Mon}: Revenue of monopoly, Rev_k: Revenue of firm in position k, *feasible: Firm profit ≥ 0

Table A.1 summarises the outcomes for the platform for various values of the auction reserve prices, for the case of asymmetric firms. The table lists the outcomes on the basis of descending value of the auction reserve price.



(a) $\alpha = 0.9$ (b) $\alpha = 1.1$

Figure A.4: Quality: Without auction

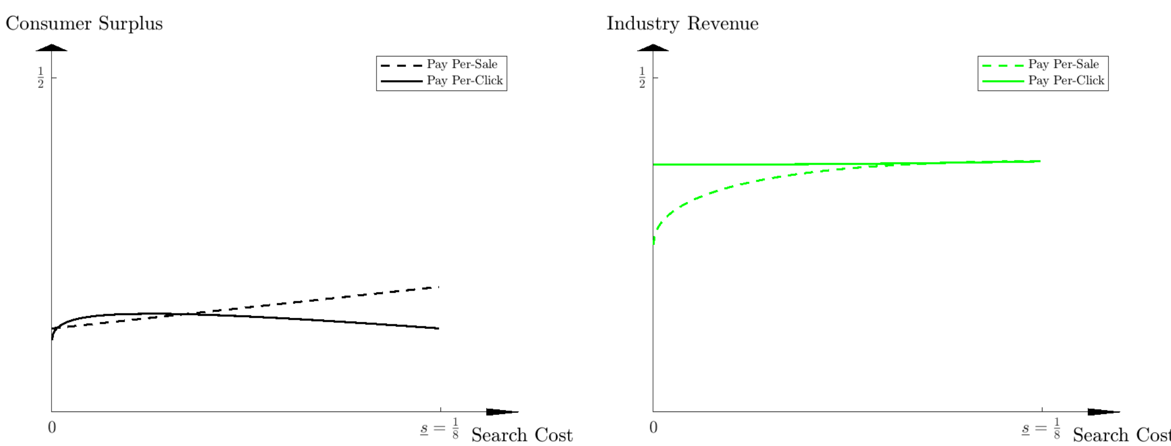


Figure A.5: Pay per-click versus Pay per-sale commission structure

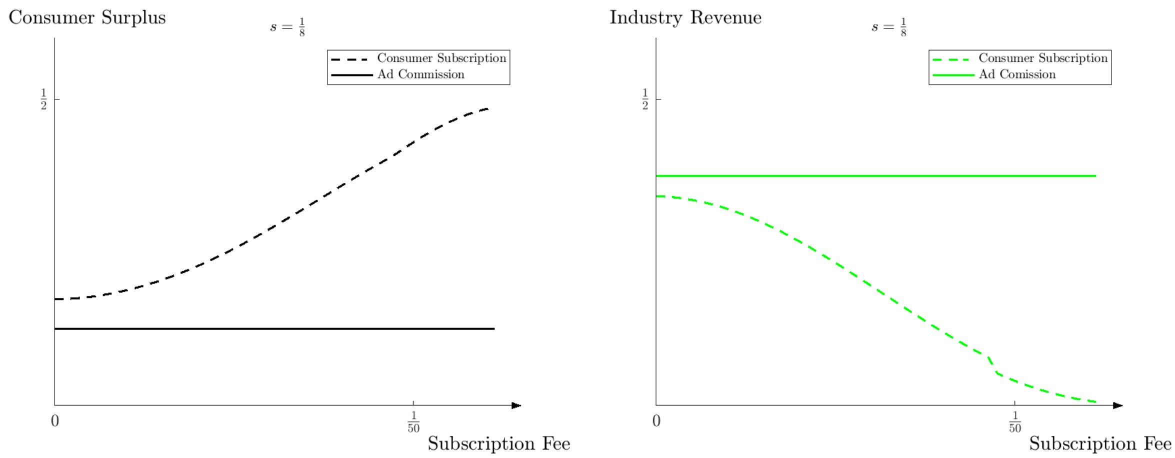


Figure A.6: Ad Commission versus Consumer Subscription

A.2 Proofs

A.2.1 Reservation value

Lemma 1. The reservation value, which determines whether a consumer buys from firm 1 without visiting firm 2 or decides to search further, is obtained by solving for the marginal consumer indifferent between searching and staying.

$$\underbrace{\mathbb{E}[v_2|v_1 - \Delta p \leq v_2 \leq \overline{v}]}_{\text{Expected benefit from visiting firm 2}} - \underbrace{(v_1 - \Delta p) \cdot [F(\overline{v}) - F(v_1 - \Delta p)]}_{\text{Oppurtunity cost of visiting firm 2}} \geq \underbrace{s}_{\text{Search cost}}$$

$$\begin{aligned} \int_{\hat{v}-\Delta p}^{\overline{v}} [(v_2 - p_2) - (\hat{v} - p_1)] f(v_2) dv_2 &= s \\ \int_{\hat{v}-\Delta p}^{\overline{v}} v_2 f(v_2) dv_2 - (\hat{v} - \Delta p)[F(\overline{v}) - F(\hat{v} - \Delta p)] &= s \\ \mathbb{E}[v_2|\hat{v} - \Delta p \leq v_2 \leq \overline{v}] - (\hat{v} - \Delta p)[F(\overline{v}) - F(\hat{v} - \Delta p)] &= s \end{aligned}$$

- $\frac{\partial \widehat{v}}{\partial s} < 0$. Using Leibniz's rule,[A.1]

$$\begin{aligned} 0 - (\hat{v} - \Delta p) f(\hat{v} - \Delta p) \frac{\partial \hat{v}}{\partial s} + 0 \; - \; \frac{\partial \hat{v}}{\partial s} [1 - F(\hat{v} - \Delta p)] & \\ -(\hat{v} - \Delta p) \left[-f(\hat{v} - \Delta p) \frac{\partial \hat{v}}{\partial s} \right] &= 1 \\ \frac{\partial \hat{v}}{\partial s} \left[-(\hat{v} - \Delta p) f(\hat{v} - \Delta p) - [1 - F(\hat{v} - \Delta p)] + (\hat{v} - \Delta p) f(\hat{v} - \Delta p) \right] &= 1 \end{aligned}$$

$$\frac{\partial \hat{v}}{\partial s} = -\frac{1}{1 - F(\hat{v} - \Delta p)} < 0$$

- $\frac{\partial \widehat{v}}{\partial p_1} > 0$. Using Leibniz's rule,

$$\begin{aligned} 0 - (\hat{v} - \Delta p) f(\hat{v} - \Delta p) (\frac{\partial \hat{v}}{\partial p_1} - 1) + 0 \; - \; (\frac{\partial \hat{v}}{\partial p_1} - 1)[1 - F(\hat{v} - \Delta p)] & \\ -(\hat{v} - \Delta p) \left[-f(\hat{v} - \Delta p)(\frac{\partial \hat{v}}{\partial p_1} - 1) \right] &= 0 \\ \left(\frac{\partial \hat{v}}{\partial p_1} - 1 \right) \left[-(\hat{v} - \Delta p) f(\hat{v} - \Delta p) - [1 - F(\hat{v} - \Delta p)] + (\hat{v} - \Delta p) f(\hat{v} - \Delta p) \right] &= 0 \end{aligned}$$

$$\frac{\partial \hat{v}}{\partial p_1} = 1 > 0$$

- $\frac{\partial \widehat{v}}{\partial p_2} < 0$. Using Leibniz's rule,

$$\begin{aligned} 0 - (\hat{v} - \Delta p) f(\hat{v} - \Delta p) (\frac{\partial \hat{v}}{\partial p_2} + 1) + 0 \; - \; (\frac{\partial \hat{v}}{\partial p_2} + 1)[1 - F(\hat{v} - \Delta p)] & \\ -(\hat{v} - \Delta p) \left[-f(\hat{v} - \Delta p)(\frac{\partial \hat{v}}{\partial p_2} + 1) \right] &= 0 \\ \left(\frac{\partial \hat{v}}{\partial p_1} + 1 \right) \left[-(\hat{v} - \Delta p) f(\hat{v} - \Delta p) - [1 - F(\hat{v} - \Delta p)] + (\hat{v} - \Delta p) f(\hat{v} - \Delta p) \right] &= 0 \end{aligned}$$

[A.1] When both the function $f(x,t)$ and its partial derivative $f_x(x,t)$ are continuous in both x and t,

$$\frac{d}{dx} \left(\int_{a(x)}^{b(x)} f(x,t) dt \right) = f(x, b(x)) \cdot \frac{d}{dx} b(x) \; - \; f(x, a(x)) \cdot \frac{d}{dx} a(x) \; + \; \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t) dt$$

$$\frac{\partial \hat{v}}{\partial p_2} \;=\; -1 \;<\; 0$$

■

Given $v_k \sim U[0, V]$.

$$l(\hat{v}, s, \mathbf{p}) \;=\; 0 \;=\; \int_{\hat{v}-\Delta p}^{V} (v_2 - \hat{v} + \Delta p) f(v_2) dv_2 \;-\; s \qquad [f(v_2) > 0 \implies 0 < \hat{v} - \Delta p < V]$$

$$\implies \frac{1}{V}\left[\frac{v_2^2}{2} - \hat{v}v_2 + \Delta p v_2\right]_{\hat{v}-\Delta p}^{V} \;-\; s \;=\; 0$$

$$\implies \frac{1}{V}\left(\frac{V^2}{2} - \hat{v}V + \Delta p V + \frac{\hat{v}^2}{2} - \Delta p \hat{v} + \frac{\Delta p^2}{2}\right) \;-\; s \;=\; 0$$

$$\implies \frac{\hat{v}^2}{2} - \hat{v}(V + \Delta p) + \frac{V^2}{2} + \Delta p V + \frac{\Delta p^2}{2} - Vs \;=\; 0$$

Quadratic roots:

$$\hat{v} \;=\; V + \Delta p \;\pm\; \sqrt{(V + \Delta p)^2 \;-\; 2\left(\underbrace{\frac{V^2}{2} + \Delta p V + \frac{\Delta p^2}{2}}_{\left(\frac{V+\Delta p}{\sqrt{2}}\right)^2} - Vs\right)}$$

$$\implies \hat{v} \;=\; V + \Delta p \;\pm\; \sqrt{2Vs}$$

Deriving constraints for $0 < \hat{v} < V$, we get

$$\hat{v} \;=\; \begin{cases} V + \Delta p \;-\; \sqrt{2Vs} & \text{when } \Delta p \in [\sqrt{2Vs} - V, \sqrt{2Vs}] \\ V + \Delta p \;+\; \sqrt{2Vs} & \text{when } \Delta p \in [-\sqrt{2Vs} - V, -\sqrt{2Vs}] \end{cases}$$

Since the two constraints on Δp do not overlap for non-zero values of s, V, we have a unique value for $\hat{v}$ when either of the constraints are satisfied.

But applying the constraint $0 < \hat{v} - \Delta p < V$, we only have

$$\hat{v} \;=\; V + \Delta p \;-\; \sqrt{2Vs} \text{ when } \Delta p \in [\sqrt{2Vs} - V, \sqrt{2Vs}]$$

A.2.2 Consumer Demand

Lemma 2. For D_{11},

$$\frac{\partial D_{11}}{\partial s} = -f(\hat{v})\frac{\partial \hat{v}}{\partial s} = \frac{f(\hat{v})}{1 - F(\hat{v} - \Delta p)} > 0$$

For D_{12}, using Leibniz's rule,

$$\begin{aligned} \frac{\partial D_{12}}{\partial s} &= F(\hat{v} - \Delta p) f(\hat{v}) \frac{\partial \hat{v}}{\partial s} - 0 + 0 \\ &= -\frac{F(\hat{v} - \Delta p) f(\hat{v})}{1 - F(\hat{v} - \Delta p)} < 0 \end{aligned}$$

For D_{22}, using Leibniz's rule,

$$\begin{aligned} \frac{\partial D_{22}}{\partial s} &= f(\hat{v})\frac{\partial \hat{v}}{\partial s} - \left[F(\hat{v} - \Delta p) f(\hat{v}) \frac{\partial \hat{v}}{\partial s} - 0 + 0 \right] \\ &= \frac{-f(\hat{v}) + F(\hat{v} - \Delta p) f(\hat{v})}{1 - F(\hat{v} - \Delta p)} < 0 \end{aligned}$$

■

A.2.3 Equilibrium - No Auction

Random Search: Solving for symmetric eq.

$$\max_p pD(p) = p\frac{D_{11} + D_{12} + D_{22}}{2}$$

For some deviation p and equilibrium price p^{Ran}

$$\begin{aligned} \frac{\partial \pi^{Ran}(p^{Ran})}{\partial p} &= \frac{\partial}{\partial p}\left(p(1 - pp^{Ran})\right) \\ &= 1 - 2(p^{Ran})^2 = 0 \\ \implies p^{Ran} &= \frac{1}{\sqrt{2}} \end{aligned}$$

It requires $pf'(p) < 2f(p)$ (or hazard rate is increasing in p) for p^{Ran} to be a global maximum.

Lemma 3. Solving for firm 1 when prices are symmetric,

$$p^* = \sqrt{2+2s} - 1$$

Solving for firm 2 when prices are symmetric,

$$p^* = \sqrt{2-2s} - 1$$

They are equal only if $s = 0$. ■

Theorem 1. Solving First order conditions, we get

$$p^*_{1,NA} = \frac{1 - F(\widehat{v}^*) + \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1}{f(\widehat{v}^*) - \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1}$$

$$p^*_{2,NA} = \frac{A \cdot [1 - F(p^*_{2,NA})]F(p^*_{1,NA}) + F(\widehat{v}^*) - F(p^*_{1,NA}) - \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1}{A \cdot f(p^*_{2,NA})F(p^*_{1,NA}) + f(\widehat{v}^*) - F(p^*_{2,NA})f(p^*_{1,NA}) - \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1}$$

For $p^*_{1,NA}, p^*_{2,NA}$ to be global maxima, we require

$$p_1 \cdot \left(f'(\widehat{v}) - \int_{p_1}^{\widehat{v}} f(v_1 - \Delta p) \cdot f''(v_1) \cdot dv_1 \right) \leq 2 \left(f(\widehat{v}) - \int_{p_1}^{\widehat{v}} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right)$$

$$p_2 \cdot \left(A \cdot f'(p^*_{2,NA})F(p^*_{1,NA}) - f'(\widehat{v}) - f(p_2)f(p_1) + F(\widehat{v} - \Delta p) \cdot f'(\widehat{v}) - \int_{p_1}^{\widehat{v}} f(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right)$$
$$\leq 2 \left(A \cdot f(p^*_{2,NA})F(p^*_{1,NA}) + f(\widehat{v}) - F(p_2)f(p_1) - \int_{p_1}^{\widehat{v}} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right)$$

■

Proposition 1. To find demand:

$$D_{11} = 1 - \frac{\widehat{v}}{V}$$

D_{12} requires $p_1 < v_1 < \widehat{v}, \ \ v_2 - p_2 < v_1 - p_1$.

$$\begin{aligned} D_{12} &= \int_{p_1}^{\widehat{v}} F_2(v_1 - \Delta p)\ f(v_1)\ dv_1 \\ &= \frac{(\widehat{v}_1 - \Delta p)^2}{2V^2} - \frac{p_2^2}{2V^2} \end{aligned}$$

D_{22} requires $v_1 < \widehat{v}, \ \ v_2 - p_2 > \max\{0, v_1 - p_1\}$.

$$D_{22} = \underbrace{\left(1 - \frac{p_2}{V}\right)\frac{p_1}{V} \cdot \mathbb{1}_{p_2 < \frac{V}{2} - s}}_{v_1 < p_1} + \underbrace{\int_{p_1}^{\widehat{v}} [1 - F_2(v_1 - \Delta p)]\ f(v_1)\ dv}_{v_1 > p_1}$$

$$= \left(1 - \frac{p_2}{V}\right)\frac{p_1}{V} + \frac{(\widehat{v} - p_1)}{V} - \frac{(\widehat{v} - \Delta p)^2}{2V^2} + \frac{p_2^2}{V^2}$$

Profit Maximisation:

$$\begin{aligned} p_1 &= \frac{V}{4} + \frac{s}{2} + \frac{p_2}{2} - \frac{p_2^2}{4V} \\ p_2 &= \frac{1}{3}\left(2p_1 + 2V - \sqrt{10p1^2 + 2p1V + 6sV + V^2}\right) \end{aligned}$$

- At $\underline{s}_{NA}$, $\hat{v} = p^*_{1,NA}$. Solving for s using the equations for prices, we get $\underline{s}_{NA} = \frac{1}{4}$.
-

$$\frac{dp^*_{1,NA}}{ds} = \frac{1 + (1 - p^*_{2,NA})\frac{dp^*_{2,NA}}{ds}}{2}$$

$$\frac{dp^*_{2,NA}}{ds} = \frac{(1 - p^*_{2,NA})\left(-1 + \frac{dp^*_{1,NA}}{ds}\right)}{\underbrace{2(1 + \Delta p^*_{NA})^2 - D^*_{22}}_{>0}}$$

Note that $(1 - p_2) \geq 1 - p^{Ran}$ and $D_{22} \leq \frac{1-(p^{Ran})^2}{2}$ whose bounds are realised when there is random search. This implies that the denominator of the RHS of the above expression is bounded below by zero.

Substituting this result in the previous expression, we get

$$\frac{dp^*_{1,NA}}{ds} = \frac{1 + (1 - p^*_{2,NA})\frac{(1-p^*_{2,NA})\left(-1+\frac{dp^*_{1,NA}}{ds}\right)}{2(1+\Delta p^*_{NA})^2 - D^*_{22}}}{2}$$

Note that

$$\frac{(1 - p^*_{2,NA})^2}{2(1 - p^*_{2,NA})^2 - D^*_{22}} = \frac{1}{2 - \underbrace{\frac{D^*_{22}}{(1 - p^*_{2,NA})^2}}_{\in[0,1)}} < 1$$

$$\implies \frac{dp^*_{1,NA}}{ds} \in (0,1) \;\; , \;\; \frac{dp^*_{2,NA}}{ds} < 0$$

- Since prices are continuous and p_1 is monotonically increasing while p_2 is monotonically decreasing, they cross each other at one point atmost (which happens at $s = 0$).
- At the lower bound, $s = 0$, oredered consumer search is equivalent to that of random search as shown below. At $s = 0$,

$$p^*_{1,NA} = p^*_{2,NA} = \frac{1 - (p^*_{NA})^2}{2} = p^{Ran} = \sqrt{2} - 1$$

At $p^*_{2,NA} = 0$, $F(\hat{v}^*) - F(p^*_{1,NA}) - \int_{p_1}^{\hat{v}^*} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1 = 0$. This implies that

$$p^*_{1,NA} = 1 - p^*_{1,NA} = p^{Mon} = \frac{1}{2}$$

This upper bound for $p^*_{1,NA}$ is hit at $(\underline{s})_{NA} = \frac{1}{4}$. For $s > \underline{s}$, condition A binds

$$p_2 < 1 - \sqrt{2s}$$

Therefore, $\overline{s} = \frac{1}{2}$.

$\blacksquare$

A.2.4 GSP Auction

Symmetric bids

Symmetric Bidding. Let η_k denote the total commission paid by firm k. Assume there exists a symmetric equilibrium where $b_i = b_j = b_{sym}$. In this case, each firm gets the first position with probability. Profit for each firm is

$$\pi_i \;=\; \pi_j \;=:\; \pi \;=\; \frac{Rev_1 + Rev_2}{2} \;-\; \frac{\eta_1 + \eta_2}{2}$$

where

$$\begin{aligned} \eta_1 &= b_{sym} \\ \eta_2 &= \widehat{b}(1 - D_{11}) \end{aligned}$$

Individual Rationality: Firms need to make non-negative profits.

$$\begin{aligned} & \pi \;\geq\; 0 \\ \iff \quad & b_{sym} \;\leq\; Rev_1 + Rev_2 - \widehat{b}(1 - D_{11}) \end{aligned} \tag{13}$$

Therefore, $b_{sym} \in [\widehat{b},\ Rev_1 + Rev_2 - \widehat{b}(1 - D_{11})]$. For any value in this set, consider a deviation.

Case One: Let $b_i > b_j = b_{sym}$ be the bids of the two firms. Assume that the positions assigned are $i \rightarrow 1,\ j \rightarrow 2$.

Individual Rationality: Firm needs to make non-negative profit.

$$\begin{aligned} & \pi_1 \;\geq\; 0 \\ \iff \quad & Rev_1 \;\geq\; b_j \end{aligned}$$

Incentive Compatibility: Firm deviates only if there is a gain from doing so.

$$
\begin{aligned}
\pi_1 \;&\geq\; \pi \;=\; \frac{\pi_1 + \pi_2}{2} \\
\iff \quad \frac{\pi_1}{2} \;&\geq\; \frac{\pi_2}{2} \\
\iff \quad \pi_1 \;&\geq\; \pi_2 \\
\iff \quad Rev_1 - b_j \;&\geq\; Rev_2 - \widehat{b}(1 - D_{11}) \\
\iff \quad b_j \;&\leq\; Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big) \qquad (14)
\end{aligned}
$$

Combining (13) and (14), there exists a deviation if

$$
\begin{aligned}
Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big) \;&>\; Rev_1 + \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big) \\
\iff \quad Rev_2 - \widehat{b}(1 - D_{11}) \;&<\; 0 \\
\iff \quad \widehat{b} \;&>\; \frac{Rev_2}{1 - D_{11}}
\end{aligned}
$$

This gives the sufficient condition to find a deviation. Then, we see that there exists no deviation of the kind $b_i > b_j$ iff

$$
b_j \;\in\; \Big[Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big)\;,\; Rev_1 + \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big)\Big] \quad \text{and} \quad \widehat{b} \;\leq\; \frac{Rev_2}{1 - D_{11}} \qquad (15)
$$

Case Two: Consider the deviation $b_i < b_j$ and position assignments $i \longrightarrow 2,\; j \longrightarrow 1$. Following similar steps, we find that there exists no such deviation iff

$$
b_j \;\in\; \Big[0, Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big)\Big) \qquad \text{and} \qquad \widehat{b} \;\leq\; \frac{Rev_2}{1 - D_{11}} \qquad (16)
$$

Combining (15), and (16), we get the optimal symmetric bid

$$
b_i = b_j \;=\; Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big) \qquad (17)
$$

This is feasible only when

$$
\widehat{b} \;\leq\; \frac{Rev_2}{1 - D_{11}} \qquad (18)
$$

Asymmetric bids

Consider the bids $b_i > b_j$ and position assignments $i \to 1,\ j \to 2$.

Individual Rationality: Firms need to make non-negative profits.

$$\begin{aligned} \pi_1 &\geq 0 \\ \iff \quad b_j &\leq Rev_1 \qquad (19) \\ \pi_2 &\geq 0 \\ \iff \quad \widehat{b} &\leq \frac{Rev_2}{1 - D_{11}} \qquad (20) \end{aligned}$$

Incentive Compatibility: Firm i deviates to position 2 iff there is a gain from doing so.

$$\begin{aligned} Rev_1 - \eta_1 &\geq Rev_2 - \eta_2 \\ \iff \quad Rev_1 - b_j &\geq Rev_2 - \widehat{b}(1 - D_{11}) \\ \iff \quad b_j &\leq Rev_1 - (Rev_2 - \widehat{b}(1 - D_{11})) \qquad (21) \end{aligned}$$

Firm j doesn't want position 1, given the above bids.

$$\begin{aligned} Rev_1 - \widetilde{\eta}_1 &\leq Rev_2 - \eta_2 \\ \iff \quad Rev_1 - b_i &\leq Rev_2 - \widehat{b}(1 - D_{11}) \\ \iff \quad b_i &\geq Rev_1 - (Rev_2 - \widehat{b}(1 - D_{11})) \qquad (22) \end{aligned}$$

Combining (21) and (22), we get the optimal asymmetric bids

$$b_i \in \left(Rev_1 - \left(Rev_2 - \widehat{b}(1 - D_{11})\right)\ ,\ \infty\right) \qquad (23)$$

$$b_j \in \left[\widehat{b}\ ,\ \ Rev_1 - \left(Rev_2 - \widehat{b}(1 - D_{11})\right)\right] \qquad (24)$$

This is feasible only when

$$\widehat{b} \leq \min\left\{\frac{Rev_1 - Rev_2}{D_{11}}, \frac{Rev_2}{1 - D_{11}}\right\} \qquad (25)$$

Auction reserve price

The intermediary's objective is to maximise its revenue.

$$\max_{\widehat{b}} \; Rev_m \;=\; b_j + \widehat{b}(1 - D_{11})$$

Note that

$$\frac{\partial \left[Rev_1 - \left(Rev_2 - \widehat{b}(1 - D_{11}) \right) \right]}{\partial \widehat{b}} \;\geq\; 0$$

This implies that b_j would take larger values when $\widehat{b}$ increases for during both symmetric and asymmetric bidding.

$$\frac{\partial Rev_m}{\partial \widehat{b}} \;\geq\; 0$$

Theorem 2. Prices are obtained by solving the first-order conditions for equation 7. For p_1^*, p_2^* to be global maxima:

$$p_1 \cdot \left(f'(\widehat{v}) - \int_{p_1}^{\widehat{v}} f(v_1 - \Delta p) \cdot f''(v_1) \cdot dv_1 \right) \leq 2 \left(f(\widehat{v}) - \int_{p_1}^{\widehat{v}} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right)$$

$$p_2 \cdot \left(A \cdot f'(p_{2,NA}^*) F(p_{1,NA}^*) - f'(\widehat{v}) - f(p_2) f(p_1) + F(\widehat{v} - \Delta p) \cdot f'(\widehat{v}) - \int_{p_1}^{\widehat{v}} f(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right)$$

$$\leq 2 \left(A \cdot f(p_{2,NA}^*) F(p_{1,NA}^*) f(\widehat{v}) - F(p_2) f(p_1) - \int_{p_1}^{\widehat{v}} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right) \;-\; r_2 f'(\widehat{v})$$

■

Corollary 2.

$$\frac{dp_1^*}{ds} > \frac{dp_{1,NA}^*}{ds} \quad , \quad \frac{dp_2^*}{ds} > \frac{dp_{2,NA}^*}{ds}$$

■

A.2.5 Asymmetric Firms

Theorem 3. There exists no symmetric equilibrium as the conditions on b_i, b_j cover disjoint intervals in $\mathbb{R}$. For asymmetric equilibrium, consider the bids $b_i > b_j$ and position assignments $i \rightarrow 1,\; j \rightarrow 2$.

Individual Rationality: Firms need to make non-negative profits.

$$\pi_{1,i} \geq 0$$
$$\iff \quad b_j \leq Rev_{1,i} \tag{26}$$
$$\pi_{2,j} \geq 0$$
$$\iff \quad \widehat{b} \leq \frac{Rev_{2,j}}{1 - D_{11,i}} \tag{27}$$

Incentive Compatibility: Firm i deviates to position 2 only if there is a gain from doing so.

$$Rev_{1,i} - \eta_{1,i} \geq Rev_{2,i} - \eta_{2,i}$$
$$\iff \quad Rev_{1,i} - b_j \geq Rev_{2,i} - \widehat{b}(1 - D_{11,j})$$
$$\iff \quad b_j \leq Rev_{1,i} - (Rev_{2,i} - \widehat{b}(1 - D_{11,j})) \tag{28}$$

Firm j doesn't want position 1, given the above bids when

$$Rev_{1,j} - \widetilde{\eta}_{1,j} \leq Rev_{2,j} - \eta_{2,j}$$
$$\iff \quad Rev_{1,j} - b_i \leq Rev_{2,j} - \widehat{b}(1 - D_{11,i})$$
$$\iff \quad b_i \geq Rev_{1,j} - (Rev_{2,j} - \widehat{b}(1 - D_{11,i})) \tag{29}$$

Combining (28) and (29), we get the optimal asymmetric bids

$$b_i \in \left(Rev_{1,j} - \left(Rev_{2,j} - \widehat{b}(1 - D_{11,i})\right) \ , \ \infty\right) = \left(Rev_{1,j} - \pi_{2,j}, \ \infty\right) \tag{30}$$
$$b_j \in \left[\widehat{b} \ , \ Rev_{1,i} - \left(Rev_{2,i} - \widehat{b}(1 - D_{11,j})\right)\right] = \left[\widehat{b}, \ Rev_{1,i} - \pi_{2,i}\right] \tag{31}$$

This is feasible only when

$$\widehat{b} \leq \frac{Rev_{1,i} - Rev_{2,i}}{D_{11,j}} \tag{32}$$

■

B Appendix: Empirical Analysis

Criteo is one of the largest players[B.1] in the e-commerce marketing sector. It works with digital platforms and retailers/brands to help serve online advertisements to consumers. Some of its main functions are to design bidding strategies for retailers in order to procure display positions, provide infrastructure to intermediaries in order to conduct real-time auctions, etc. Criteo continuously collects a large amount of data on consumers' online behaviour and firms' advertisement characteristics to understand when searchers convert to buyers and how firms bid.[B.2] Hence, this user-level data provides a unique opportunity to explore the interaction between consumers and firms in the ad auction market.

B.1 Figures and Tables

Table B.1: Heterogeneity: by user engagement

Outcome: commission	(1)	(2)	(3)	(4)
low user engagement (by clicks)	-0.000193***		-0.000143***	
	(0.000)		(0.000)	
low user engagement (by purchases)		-0.000329***		-0.000279***
		(0.000)		(0.000)
prominence			5.11e-05***	9.49e-05*
			(0.000)	(0.000)
prominence × low user engagement (by clicks)			-9.03e-05***	
			(0.000)	
prominence × low user engagement (by purchases)				-9.30e-05*
				(0.000)
Observations	1,536,034	1,592,119	1,536,034	1,592,119
R-squared	0.052	0.051	0.052	0.051
mean(y)	0.000192	0.000192	0.000192	0.000192
sd(y)	0.000540	0.000540	0.000540	0.000540
ad FE	✓	✓	✓	✓
day of the week FE	✓	✓	✓	✓
hour of the day FE	✓	✓	✓	✓

[B.1] Criteo generated close to $1B in annual revenue in 2017 in USA, the year in which this data was released. (International Data Corporation (IDC) report 2017. Accessed May 18, 2021)

[B.2] Criteo has collected data on more than 2.5B users and 3500 products, by 2021. (https://criteo.investorroom.com, accessed May 18, 2021.)

Table B.2: Evidence on Equilibrium Refinement

$\mathbb{E}[y_1 \cdot y_2]$	0.388***
	(0.0003)
$\mathbb{E}[y_1] \cdot \mathbb{E}[y_2]$	0.286***
	(0.0002)
$\mathbb{E}[y_1 \cdot y_2] - \mathbb{E}[y_1] \cdot \mathbb{E}[y_2]$	0.102***
	(0.0004)

Notes. Controls: Ad-pair, day of the week, hour of the day.

where y_i denotes the equilibrium commission for firm in position i (1: prominent, 2: non-prominent).

Table B.3: Robustness: Bidding strategy not driven by expected engagement

(a) beginning of month (higher engagement)

$\mathbb{E}[y_1 \cdot y_2]$	0.387***
	(0.0003)
$\mathbb{E}[y_1] \cdot \mathbb{E}[y_2]$	0.293***
	(0.0003)
$sgn(\mathbb{E}[y_1 \cdot y_2] - \mathbb{E}[y_1] \cdot \mathbb{E}[y_2])$	0.094***
	(0.0005)

(b) end of month (lower engagement)

$\mathbb{E}[y_1 \cdot y_2]$	0.360***
	(0.0006)
$\mathbb{E}[y_1] \cdot \mathbb{E}[y_2]$	0.312***
	(0.0007)
$sgn(\mathbb{E}[y_1 \cdot y_2] - \mathbb{E}[y_1] \cdot \mathbb{E}[y_2])$	0.048***
	(0.001)

Notes. Controls: Ad-pair, day of the week, hour of the day. Motivated by the fall in purchases at the end of month as plotted in figure 9, I define the end of the month as days 25 to 31.

For online publication: Appendix of *Sponsored Search: How Platforms Exacerbate Product Market Concentration*

A Price Visibility

Equilibrium prices in a market where consumers are free to observe prices prior to visiting a firm's webpage (*in-sight*) are lower than the equilibrium prices in the same market with *out-of-sight* prices, *ceteris paribus*. Intuitively, a seller's demand is more elastic with respect to changes in *in-sight* prices than when price is only discovered after the consumer pays the search cost. Consider the unit mass of consumers plotted in Figure A.1. The co-ordinates of the square classify consumers by their valuation of the two products. For a small change in p_2, we see that demand for firm 2 changes along both line segments LM and MN. Since firm 1 shares those line segments, these consumers are taken over by it. On the other hand, imagine a situation where prices are *out-of-sight*. For a small change in p_2, we see that demand for firm 2 changes along line segment LM only. In other words, demand for firm 2 is now less elastic compared to the case of *in-sight* prices. This reduction in elasticity is sufficient to overturn the order of prices.

In models of random search (Wolinsky, 1986, Anderson and Renault, 1999) and prominence (Armstrong, Vickers, and Zhou, 2009), firms might lower prices not to attract consumers, but to retain them once they visit. In my model, prices serve both roles: they help to both attract and retain consumers.[A.1] Specifically, a lower price for firm 2 not only retains more visitors, but is also more likely to attract them; the latter effect makes its demand more elastic. This suggests that when consumers observe prices before searching, prices decrease with search costs. This can be observed from the price of firm 2. Since price of firm 1 is observed costlessly by the consumer at the beginning of search, this effect is absent in its case. Therefore, price dispersion increases with search cost in my model.

To further understand effect of *in-sight* prices, I disentangle the channels which influence firm 2's decision. When consumers visit firm 2, it learns that consumers, on average, have a lower valuation of product 1. This makes those consumers more likely to stay at firm 2, thus applying an upward force on p_2. I refer to this as the *information channel*.

When prices are *out-of-sight*, consumers visit firm 2 based on their rational belief about its price. However, it is free to surprise them as it doesn't change their incentives to visit firm 2. This also applies an upward force on p_2. I refer to this as the *hold-up channel*. Note that, the case of *out-of-sight* prices influences through both channels. However, making prices free to observe eliminates firm 2's ability to surprise and shuts down the *hold-up channel*. Therefore, I find that only having the information channel is not sufficient to generate increasing order of prices.

To illustrate this effect, I consider a market where $\lambda \in [0, 1]$ fraction of consumers can see both prices before starting their search while the rest discover prices by search. This exercise nests

[A.1] Some previous studies consider prices as free to observe under different conditions. For instance, Dai (2017) uses it to analyse limited commitment and Arbatskaya (2007) uses it to analyse heterogeneous search cost.

equilibrium prices from my benchmark model at $\lambda = 1$ and those from the Armstrong, Vickers, and Zhou (2009) (AVZ) model (which predicts $p_1 < p_2$) at $\lambda = 0$.

Figure A.2 shows that as prices become free to observe to more consumers (increase in λ), the role of p_2 in attracting consumers gains significance. The *hold-up* force, which had caused higher prices for firm 2 in AVZ, is weaker now due to an increase in the price elasticity of firm 2's demand. This reverses the price order.

As search cost increases, it makes the firms incentives to attract consumers more important. Therefore, even when a smaller fraction of consumers see prices before search, we see a reversal in price rankings. Moreover, this exercise of parametrizing λ resembles the 'clearing-house' models of search (see, for instance, Varian, 1980, Perloff and Salop, 1985) where a fraction of the population have access to prices. In those models, we usually see a mixed-strategy equilibrium. However, in my model, the underlying consumer heterogeneity allows for the existence of pure-strategy equilibrium.

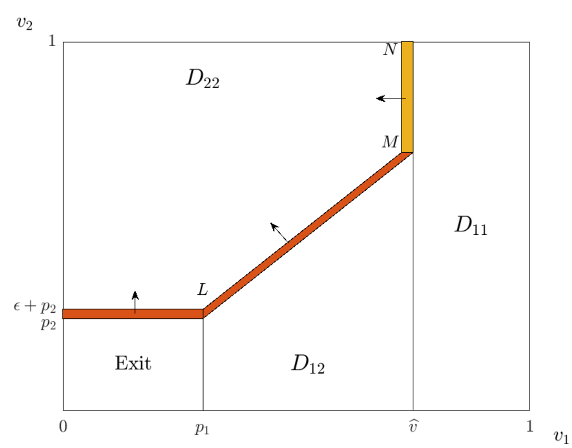


Figure A.1: Elasticity of demand: Intuition for prices

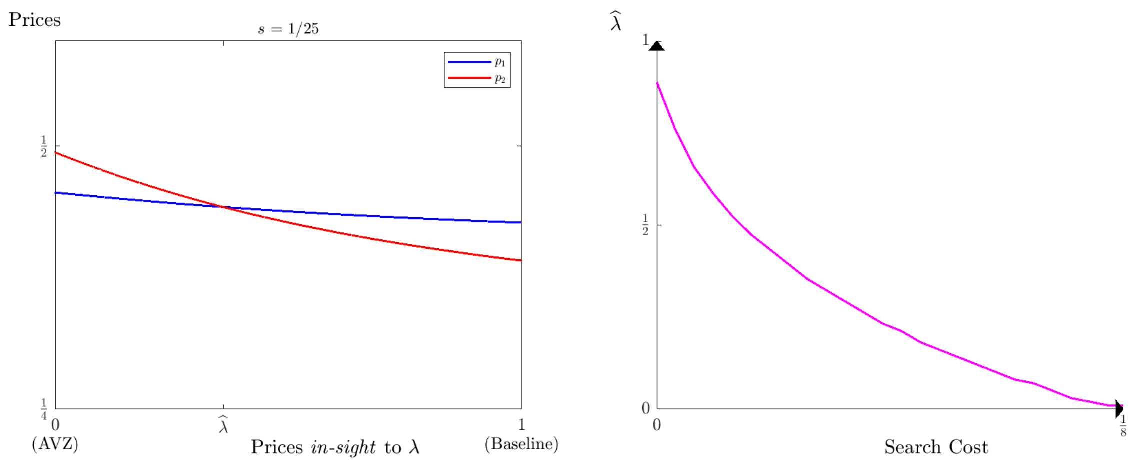


Figure A.2: Weakening of the Hold-up channel

B Revenues: 'No Auction'

In this section, I derive the equilibrium revenues for both firms, in the absence of the auction.

Proposition 5. *When match values are drawn from a Uniform distribution ($v \sim F = U[0,1]$), the equilibrium revenue for firms diverge with search cost.*

$$\frac{dRev_1^*}{ds} > 0 \ , \ \frac{dRev_2^*}{ds} < 0$$

Proof. Using Envelope Theorem and substituting the expressions for the derivatives,

$$\begin{aligned}
\frac{dRev_1^*}{ds} &= p_{1,NA}^*\left(\frac{dD_{11}^*}{ds}+\frac{dD_{12}^*}{ds}\right)\\
&= p_{1,NA}^*\left(-\frac{dp_{1,NA}^*}{ds}+(1-p_{2,NA}^*)\frac{dp_{2,NA}^*}{ds}+1\right)\\
&= p_{1,NA}^*\left(-\frac{1+(1-p_{2,NA}^*)\frac{dp_{2,NA}^*}{ds}}{2}+(1-p_{2,NA}^*)\frac{dp_{2,NA}^*}{ds}+1\right)\\
&= p_{1,NA}^*\left(\frac{1+(1-p_{2,NA}^*)\frac{dp_{2,NA}^*}{ds}}{2}\right)\\
&= p_{1,NA}^*\left(\frac{dp_{1,NA}^*}{ds}\right)\geq 0\\
\frac{dRev_2^*}{ds} &= p_{2,NA}^*\left(\frac{dD_{22}^*}{ds}\right)\\
&= p_{2,NA}^*\left(-1\underbrace{-(1-p_{2,NA}^*)\frac{dp_{2,NA}^*}{ds}}_{\leq 1 \text{ from Proposition 1}}\right)\leq 0
\end{aligned}$$

■

Figure B.1 visualises the comparative statics of firm revenues, following proposition 5. As search cost increases, there is a decrease in competition (region I). This increases the revenues as seen for firm 1 and reaches the monopoly level at $\underline{s}$. For $s > \underline{s}$, firm 1 maintains its optimal price at the monopoly level.

On the other hand, due to *in-sight* prices, firm 2 has lost its ability to extract the hold-up rent and the price decreases along with its (region I). In region II, firm 2's demand increases with s. This is because it now has a constant population of visitors to attract and retain and as it decreases its price, more consumers find the offer feasible and purchase. However, the revenues are well below the optimum due to constraint A. Therefore, revenue continues to decrease.

The industry revenue (sum of revenues) is maximum at the lowest search cost where both firms have segmented demand. This is because $\underline{s}$ would impose the loosest constraint on firm 2 while there is no competition (both firms have their own set of visitors).

Figure B.1 also compares the equilibrium industry revenues from my baseline model with the case where a single firm is selling both products. I refer to this second case as the Multi-product

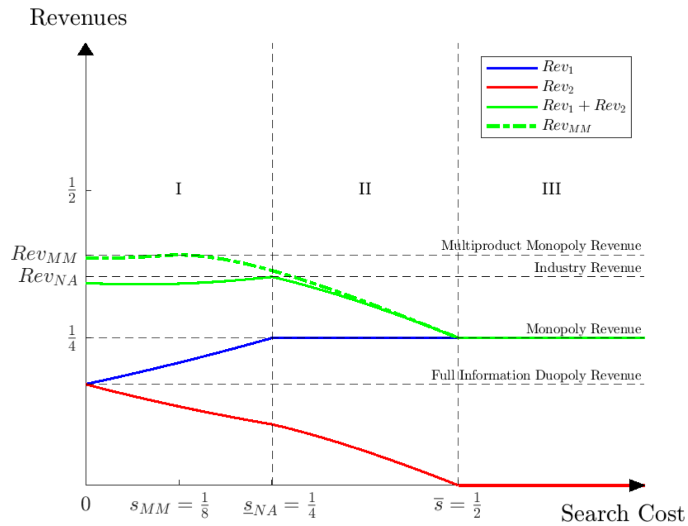


Figure B.1: Revenues (without auction)

Monopoly (MM).[B.1] Drawing parallels between the intermediary's function and MM can be helpful while considering policy tools applicable to the online market. From the perspective of the intermediary, this benchmark may also be useful to assess alternate position-allocation mechanisms.

The MM uses search cost to (imperfectly) screen consumers based on their preferences. Therefore, MM finds it optimal to set some $s > 0$. Only the consumers with lower valuations for product 1 are catered to by the second product. This lets the MM charge a higher price for product 1, $(p_1)_{MM} > (p_2)_{MM}$. The MM does not lose as many consumers - if it raises its prices - as when firms are separate because some consumers buy from the other firm now (which is also a subsidiary of MM). Hence, prices of both products are higher in the case of MM than in the my baseline model.

Figure B.1 also shows that the optimal s for the MM (s_{MM}) is lower than $\underline{s}$. This is because the MM is able to reach region II quicker. Since MM internalises the fact that some consumers that leave firm 1 for high prices buy product 2 instead, MM prices the goods more aggressively and there is a segmentation of the market for lower s. Thus, the maximum revenue for MM is also larger than the industry revenue when firms are separate.

[B.1] See, for instance, Petrikaitė (2018) and Gamp (2016). Both papers characterise the optimal search cost such that a multi-product monopoly firm chooses to maximise its revenues.

C Constant Commission

In this section, I introduce to the firms' problem an ad cost or commission. I consider the case where firms face a constant commission for any search cost.

Firms' problem. Firms maximise their profits by choosing product prices (**p**) and r_1 and r_2 are per-click commission to be paid by the respective firms. The analysis in Section 3 of the main text is a special case of this framework where $r_1 = r_2 = 0$. In Section 4 of the main text, r_1 and r_2 are endogenous objects derived from the auction. Firms maximise profit.

$$\max_{p_{1,C}} \pi_1 = p_{1,C}(D_{11} + D_{12}) - r_1(b_i, b_j, \widehat{b}) \tag{33}$$

$$\max_{p_{2,C}} \pi_2 = p_{2,C} D_{22} - r_2(b_i, b_j, \widehat{b}) \cdot (1 - D_{11}) \tag{34}$$

Subscript C denotes 'constant commission'. This framework with two firms parsimoniously allows me to highlight the differences in the cost structure of firms, due to positioning. From the above equation, we see the different roles per-click commissions play. Since all consumers visit firm 1, the ad commission plays the role of a fixed cost for firm 1, while it is analogous to a marginal cost per visitor for firm 2. This asymmetry, again a consequence of asymmetry in ad position, plays a crucial role in determining the equilibrium outcome, as shown in Proposition 6 and later in the full equilibrium with auction in Theorem 2 of the main text.

Proposition 6 (Constant commission)**.** *Under an exogenous per-click commission fees of r_1 and r_2 is imposed, there is a unique asymmetric equilibrium of prices in pure strategies and are given by*

$$p^*_{1,C} = \frac{1 - F(\widehat{v}^*) + \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f(v_1) \cdot dv_1}{f(\widehat{v}^*) - \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f'(v_1) \cdot dv_1}$$

$$p^*_{2,C} = \frac{A \cdot [1 - F(p^*_{2,C})]F(p^*_{1,C}) + F(\widehat{v}^*) - F(p^*_1) - \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f(v_1) \cdot dv_1 + r_2 f(\widehat{v}^*)}{A \cdot f(p^*_{2,C})F(p^*_{1,C}) + f(\widehat{v}^*) - F(p^*_2) f(p^*_1) - \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f'(v_1) \cdot dv_1}$$

When match values are drawn from a Uniform distribution ($v \sim F = U[0,1]$),

- *the search cost at which the "Attraction" condition starts binding is lower than the 'No auction' case and is given by $\underline{s}_C = \frac{1-2r_2}{4} < \underline{s}_{NA}$*
- *the price of prominent firm increases with search cost $\frac{dp^*_{1,C}}{ds} > 0$, and the price of non-prominent firm decreases with search cost $\frac{dp^*_{2,C}}{ds} < 0$*
- *and there exists $\widehat{s}$ such that the price difference is positive for lower search costs, that is $p^*_{1,C} -$*

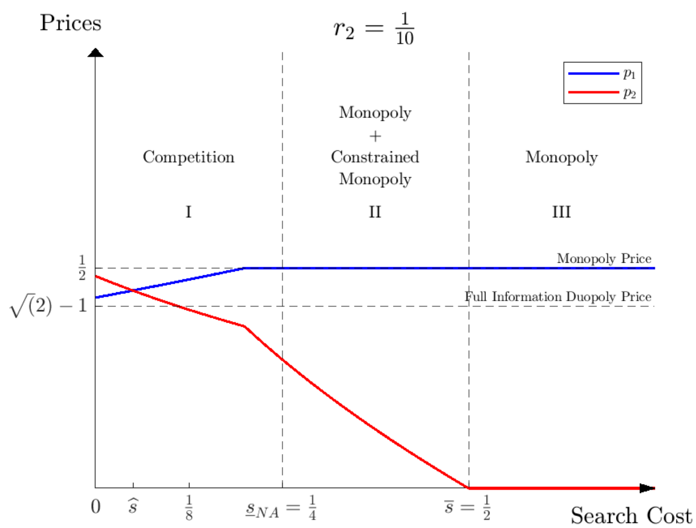


Figure C.1: Prices (with constant commission)

$$p^*_{2,C} \begin{cases} < 0 & \textit{if } s < \hat{s} \\ > 0 & \textit{if } s > \hat{s} \end{cases}$$

Proof. Prices are obtained by solving the first-order conditions for equation 33. For $(p^*_1)_C, (p^*_2)_C$ to be global maxima:

$$p_1 \cdot \left(f'(\hat{v}) - \int_{p_1}^{\hat{v}} f(v_1 - \Delta p) \cdot f''(v_1) \cdot dv_1 \right) \leq 2 \left(f(\hat{v}) - \int_{p_1}^{\hat{v}} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right)$$

$$p_2 \cdot \left(A \cdot f'(p^*_{2,NA}) F(p^*_{1,NA}) - f'(\hat{v}) - f(p_2) f(p_1) + F(\hat{v} - \Delta p) \cdot f'(\hat{v}) - \int_{p_1}^{\hat{v}} f(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right)$$
$$\leq 2 \left(A \cdot f(p^*_{2,NA}) F(p^*_{1,NA}) f(\hat{v}) - F(p_2) f(p_1) - \int_{p_1}^{\hat{v}} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right) - r_2 f'(\hat{v})$$

At $s = 0$,

$$(p^*_1)_C = \sqrt{2} - 1 = p^{Ran}$$
$$(p^*_2)_C = \sqrt{2(1 + r_2)} - 1 > (p^*_1)_C$$

■

Proposition 6 shows that for sufficiently low commission rates, there exist search costs for which $p_2 > p_1$ (see, for example, figure C.1). The marginal cost for visits for firm 2 increases its price which, in turn, gives room for firm 1 to also raise its price without the fear of losing consumers. However, the effect on firm 1 is of second-order. Note that firm 1 faces a fixed cost of commission. Hence, there is an asymmetric rise in p_2. This result reconciles with markets observed with increasing order of prices. However, the order is determined in my model due to the asymmetry in the cost structure. Thus, a non-prominent firm can charge a higher price, even when the *hold-up channel* has been eliminated.

The equilibrium prices diverge with search cost, following the properties highlighted in Section 3 of the main text. Note that $\underline{s}_C < \underline{s}_{NA}$ as prices are higher compared to the 'No Auction' case. The rise of p_2 due to marginal cost-like commission weakens competition as it damages firm 2's ability to attract and retain consumers. This gives firm 1 more power in the market for all s. Hence, p_1 shifts closer to the monopoly price and leads to a segmented market for a smaller s.

D Costly first search

For $s > \overline{s}$ (region III), the market breaks down as nobody is interested in searching. The action happens in region II.

Recall that the reservation value for a consumer considering between staying at firm 1 and visiting firm 2 is given by

$$\hat{v} = 1 + p_1 - p_2 - \sqrt{2s}$$

The market would be active only if the reservation value exceeds the outside option. For $\underline{s} < s < \overline{s}$, this constraint binds (region II) and can be written as $p_2 < 1 - \sqrt{2s}$. The only question that remains now is to determine if the firms would prefer to deviate from $p_1 = p_2 = 1 - \sqrt{2s}$ (the equilibrium prices at $s = \underline{s}$) and profits of $\pi_1 = (1 - \sqrt{2s})\sqrt{2s}$ and $\pi_2 = (1 - \sqrt{2s})^2\sqrt{2s}$.

They do not deviate to a higher price because they would then not have any visitors. Let firm 1 charge $p_1 = 1 - \sqrt{2s}$ and firm 2 deviate to $p_2 = 1 - \sqrt{2s} - \epsilon$. Then, $\widehat{v} > p_1$ and we are in the competitive framework (á la region I). Firm 1 gets a measure 1 of visitors and a measure $\sqrt{2s}$ of buyers among them. As an approximation, let me ignore the ϵ visitors that consider both firms, since their contribution to firms' demand would only be second-order. So, the remaining visit firm 2 and it gets a measure $(1 - \sqrt{2s})(\sqrt{2s} + \epsilon)$ of buyers. Then, firm 2 gets a profit of $(1 - \sqrt{2s})(1 - \sqrt{2s} - \epsilon)(\sqrt{2s} + \epsilon)$.

$$\pi_2 + (1 - 2\sqrt{2s} + 2s)\epsilon - (\sqrt{2s} - 2s)\epsilon - \epsilon^2(\cdot) \;\approx\; \pi_2 + (1 - 4\sqrt{2s} + 4s)\epsilon > \pi_2$$

Hence, it will prefer to deviate. Now, imagine firm 1 deviates instead. its profit would be $(1 - \sqrt{2s} - \epsilon)(\sqrt{2s} + D_{12})$ where $\epsilon > D_{12} = \mathbb{P}[v_1 > v_2 - \epsilon] = (\frac{1}{2} - \epsilon)$.

$$\pi_1 - \sqrt{2s}\epsilon + (\frac{1}{2} - \epsilon)(1 - \sqrt{2s} - \epsilon) \approx \pi_1 + \frac{1 - \sqrt{2s} - \epsilon}{2} - \epsilon > \pi_1$$

Hence, it also prefers to deviate. Now, imagine $p_1 = p_2 = 0$ and firm 2 deviates to $p_2 = \epsilon$. This would generate a positive profit for firm 2. Therefore, although it requires a more formal analysis, it seems like there is no equilibrium in pure-strategies for $\underline{s} < s < \overline{s}$.

E Timing of the game

In the baseline model, firms place their bids and learn their position before setting product prices. This is motivated by the intuition that the one-shot auction in my model is an approximation of a long dynamic game which has revealed all the unknown information about the auction outcome. Hence, firms set prices as if they know their position and commission. See also, for instance, Gorodnichenko and Talavera (2017) for recent evidence on the high frequency of price changes in online markets. To extend the fit of my model to a more general setting, I explore alternate timings below.

Auction reserve price → firms bid and set prices simultaneously: This case is relevant to situations where auctions are nearly as frequent as product price adjustments. In symmetric equilibrium, the firms now have to set one price for both positions they will take up with equal probability. In such situations, firms' objective can be represented by

$$\begin{aligned}
\max_{p_i} Rev_i &= p_i \cdot \frac{D_1 + D_2}{2} = p_i \cdot \frac{1 - p_i p_j}{2} \\
\text{Symmetry} \implies p &= \frac{1}{\sqrt{3}} (= p^{MM}) \\
\implies Rev &= \frac{1}{3\sqrt{3}} \approx 0.192 \\
\implies \widehat{b} &= \frac{1}{\sqrt{3}} \\
\implies Rev_m &\approx 0.385 > \max\{Rev_m^{baseline}(s)\} \\
\pi &= 0
\end{aligned}$$

Note that this outcome is not a function of search cost. The asymmetric equilibria are identical to the baseline model, since we look at subgame-perfect Nash equilibria and hence, beliefs are true in equilibrium. However, we do not have the symmetric equilibrium we derived for the baseline model.

The crucial difference here is that firms do not know their position when they are setting a price. In the symmetric equilibrium of the baseline model, even though firms get each position with equal probability (due to tie-breaking), they know their position when they set prices. For example, consider that there are 100 instances of the same two firms competing for the top spot. In the baseline model, the position-allocation is revealed to the firms before the price setting. However, this is not true when firms set prices and bid simultaneously. In each of the 100 instances, the firms have to set prices without knowing their position, even though they end up getting each position with equal probability.

Auction reserve price → firms set prices → firms bid: This case is relevant to situations where auctions are rare (for e.g., quarterly) compared to product price adjustments. Solving backwards, we know that demand at position 1 is higher, even though they are more price elastic. Both firms prefer the first position and they both bid $b = Rev^{Mon}$. Therefore, they set prices equal to $\frac{1}{\sqrt{3}}$ since they do not know their position when they set prices. The logic from the previous case applies here as well.

Firms set prices → auction reserve price → firms bid: This case is relevant to situations where ad display designs changes at a higher frequency than product prices. In other words, pricing strategies are much more stickier than advertisement strategies.

The outcome here is same as in the previous case. Exchanging the timing of setting auction reserve price does not change outcomes for subgame-perfect Nash equilibria.

F Position auction

Equilibrium refinement: The tradition of restricting the set to undominated strategies is much older and attributed to analyses of co-operative game theory where the 'core' of a game is given by the undominated outcomes (Roth and Sotomayor, 1990). If as a planner or auctioneer, one would like to advice the firms to avoid some 'bad strategies', one may prescribe playing the undominated strategies. Note that this may not necessarily lead to a unique equilibrium.

Equilibrium in undominated strategies coincides with a unique locally-envy free refinement in my model. Edelman, Ostrovsky, and Schwarz (2007) develops this refinement informed by the markets' dynamic structure. Although there is still multiplicity due to the fact that firm 1's range of bids does not have an upper bound, this does not play a role in the equilibrium outcome.

Further, Varian (2007) also independently addresses the multiplicity and illustrates two possible refinements. The paper describes the intuition behind an aggressive approach - "what is the highest bid I can set so that if I happen to exceed the bid of the agent above me and I move up by one slot, I am sure to make at make at least as much profit as I make now?" - and a defensive approach - "If I set my bid too high, I will squeeze the profit of the player ahead of me so much that it might prefer to move down to my position". The paper also derives a lower-bound of outcomes from a set of sub-game perfect Nash equilibria which coincides with the 'locally envy-free' refinement mentioned above.

Microfoundation: Standard models of position auction oversee a certain form of interdependency in object (position in this case) values across bidders (firms in this case). The value of a click for a firm is assumed to be constant across positions (see, for instance, Varian, 2009). Although it is common to assume that the click-through rate as a product of firm-specific factor times a position-specific factor, consumer behaviour remains simplified. For instance, Athey and Ellison (2011) assumes the number of clicks to be exogenous. These simplifications may not be innocuous but definitely helped gain insights on various features of the position auctions.

In my model, I derive consumer behaviour as a function of the price, which in turn, is a function of both value of a buyer and number of clicks and the commission (or the cost per click). Previous literature assumed the shape of these functions. Using consumer search as a model of consumer behaviour, we now know how these functions would look like. More importantly, now we know not just the functions but that there is a feedback, and what we want and can find is the fixed point.

Reserve Price: Edelman and Schwarz (2010) assesses the welfare effects of the auction reserve price. They separate the effects into direct (causing the lowest value bidder to face a higher payment - in my model it is firm 2) and indirect (inducing other bidders to increase their bids, thereby increasing others' payments - in my model it is firm 1). They find that most of the incremental revenue from setting a reserve price optimally comes not from a direct effect, but rather from the indirect effects on high bidders. I find a similar result that the larger share of intermediary revenue comes from firm 1 for any increment in $\widehat{b}$ (see, for instance, figure F.1). Although top bidders' large valuations place them 'furthest' from the reserve price, they contribute more as commission. This is due to two forces. First, firm 2 is now forced to charge a higher price to cover its commission cost which allows firm 1 to also increase (relatively smaller than firm 2's increase) its price without loss. Second, this makes firm 2 less attractive. Thus, overall revenues of firm 1 increases substantially, to the benefit of the intermediary. Further, characterisation of the auction reserve price taking into account endogenous consumer behaviour has shown the existence of a strategy-proof and optimal allocation mechanism.

Similarly, on the other side, the auction reserve price also has an effect on the overall conversion rate in the market and how many consumers choose outside option.

One-shot auction: Note that the assumption of one-shot simultaneous bidding is a deviation from the possibility of continuous asynchronous bidding allowed in auctions run by some

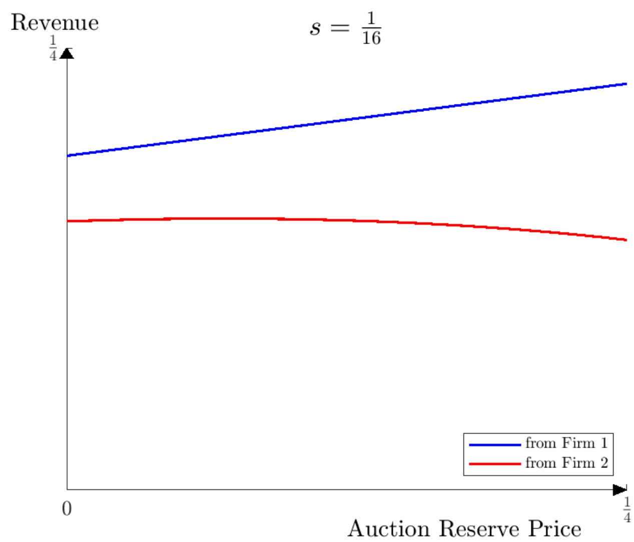


Figure F.1: Indirect effect of reserve price is stronger than direct effect

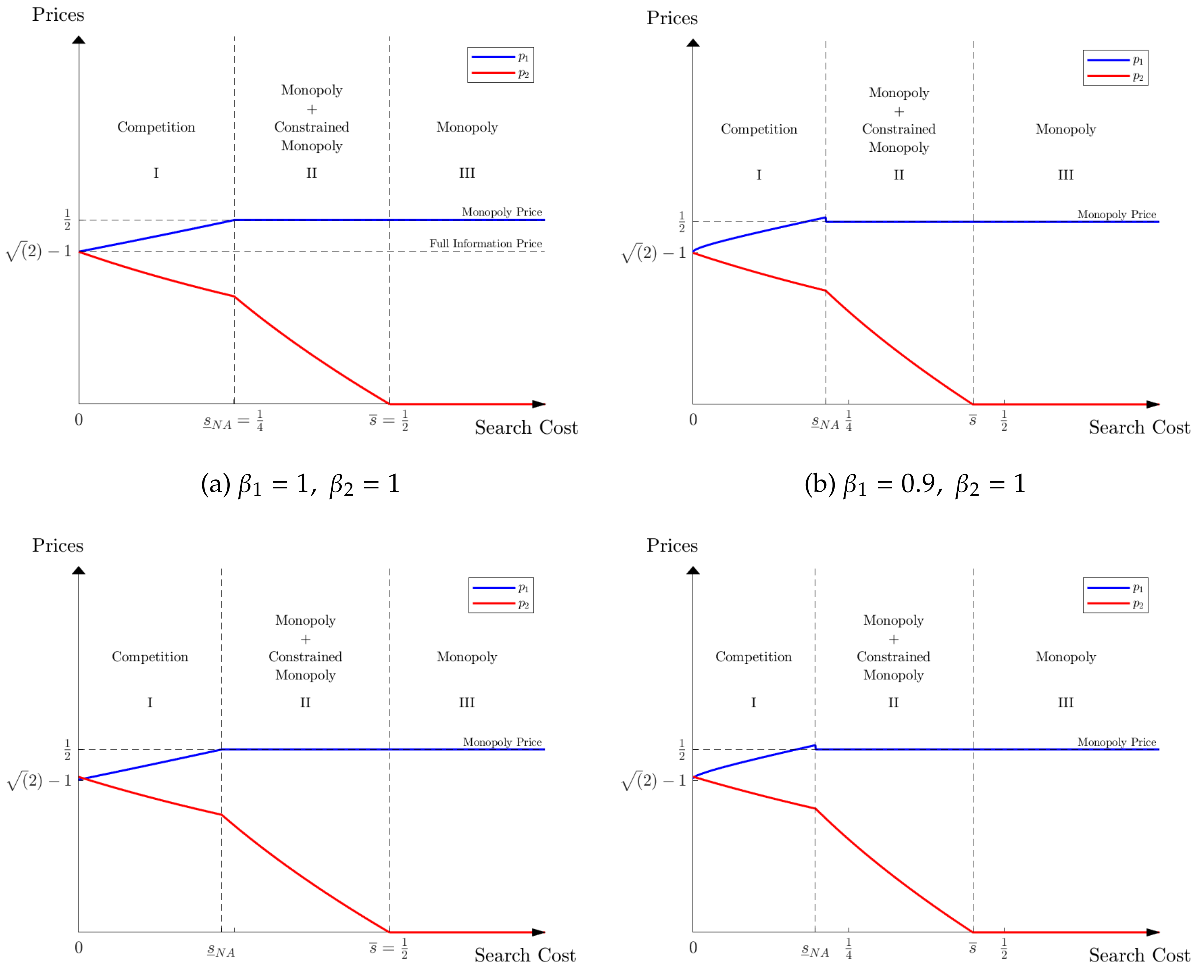
popular search engines. As in previous theoretical work on position auctions, I make this assumption for simplicity. Edelman, Ostrovsky, and Schwarz (2007) shows that the "lowest revenue envy-free" equilibrium generates the upper bound of revenues from dynamic auction models. However, having relaxed the assumption of homogenous position effects in my model, this may no longer clear be the case and requires further study. This concern is also shared in Goldman and Rao (2016). Unfortunately, an analysis of this dynamic auctions is beyond the scope of this paper.

G Asymmetric Firms: Branding/Relevance

Imagine a consumer searching for a particular brand as the keyword. Competitors of this 'focal brand' may also bid for ad positions. Consumers are more likely to visit these firms if their products are more relevant to their initial search.[G.1] However, it is *ex-ante* unclear how many of these consumers the firms will be able to convert and how their equilibrium prices will be. One can also interpret this as the intermediary targeting its display ads to a demography that is interested in a particular brand. I explore this extension of asymmetric firms in this section.[G.2]

[G.1] See, for instance, Eliaz and Spiegler (2011) and Simonov and Hill (2021).

[G.2] Another way to think about this extension is that some fraction of consumers who search for a particular brand find a competitor brand's ad to be less relevant. This kind of heterogeneity in ordering by relevance might also happen when firms misunderstand the keyword yet, mistakenly, participate. On the other hand, a platform may even do this deliberately for two reasons. One, when they have seen many consumers who enter a certain keyword but end up buying a slightly different product, or second, when they want to increase the visibility of a certain firm. I do not take a stand on this and model the reduced form.



(a) $\beta_1 = 1,\ \beta_2 = 1$

(b) $\beta_1 = 0.9,\ \beta_2 = 1$

(c) $\beta_1 = 1,\ \beta_2 = 0.9$

(d) $\beta_1 = 0.9,\ \beta_2 = 0.9$

Figure G.1: Relevance: Without auction

Let firms have relevance $\beta_1 > \beta_2$ (abuse of notation anticipates the equilibrium outcome). Let $\widetilde{D}$ denote the demand for firms when there is heterogeneous relevance. The value of product $k = \{i, j\}$ (from firm k) for each consumer is drawn from a mixture

$$\begin{cases} = \ 0 & \text{with prob. } 1 - \beta_k \\ \sim \ U[0,1] & \text{with prob. } \beta_k \end{cases}$$

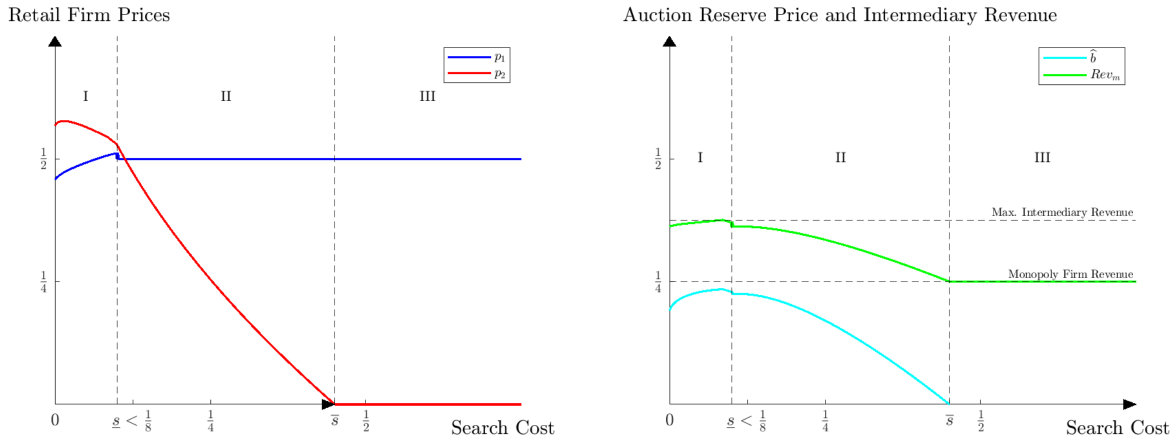


Figure G.2: Relevance ($\beta_1 = 1,\ \beta_2 = 0.9$): With auction

Deriving the reservation value for the search rule, we get

$$\tilde{v}_\beta \;=\; \beta_2 + \widetilde{\Delta p} - \sqrt{2\beta_2 s}$$

where $\widetilde{\Delta p} = p_1 - \beta_2 p_2$. Note that the effect of p_2 on $\tilde{v}_\beta$ is dampened but β_2 is now an additional characteristic about firm 2 that consumers consider before deciding whether to search. Figure G.2 plots the equilibrium results for $F = U[0,1]$.

H Alternative Auction Format

Digital platforms are constantly innovating in their technology to sell ads, especially in the type of auction conducted.[H.1] In the following exercise, I compare the baseline format of Generalised Second Price auction with the Generalised First Price auction (implemented widely by Yahoo!).[H.2] I analyse the implications on welfare when the intermediary conducts an auction and both firms *pay their own bid* as the per-click ad commission.

Proposition 7. *Symmetric pricing equilibrium does not exist (due to the tie-breaking rule). Asymmetric pricing equilibrium in pure strategies resembles that of the baseline model (Proposition 2 of the main text).*

Corollary 4. *Following Proposition 7, Proposition 4 of the main text on the equilibrium outcomes in a pay per-sale model also holds in a first-price auction setting.*

[H.1] See, for instance, CMA (2020a) and Ferrer, Ilango, and Richter (2022) for an overview of recent technological developments and Decarolis, Goldmanis, and Penta (2018) for a discussion of auction formats.

[H.2] Note that for the case of two firms, a Vickrey-Clarke-Groves auction (implemented widely by Facebook) coincides, by design, with the GSP auction (implemented widely by Google).

Firstly, there is non-existence of a symmetric equilibrium due to cyclic deviations: each firm would like to reduce its bid in order to pay less commission, but for a low enough competing bid, it would like to raise its bid to obtain the prominent position. Secondly, there is no difference between using first or second price auction from the firms' point of view, since under the symmetric bidding equilibrium, the highest bid and the second highest bid coincide. Since I resolve this symmetry using the tie-breaking rule to assign positions, the sub-game that follows is also identical.