Application Compatibility in the presence of Preference for Variety

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With the rising relevance of digital products, there has been an increased regulatory focus on understanding the competitive dynamics of digital ecosystems. Using a theoretical model, we examine competition between two multi-product firms, each selling a hardware (with advertisements in it) and an application (containing exclusive content), where each firm decides on the compatibility of its application with the rival hardware device. A novel feature of our model is that some consumers have a preference for consuming a variety of applications. Our analysis shows that both application and hardware prices are highest when both firms make their application compatible with rival hardware; whereas application prices are lowest when both firms make their application incompatible with rival hardware, and hardware prices are lowest when one firm makes its application compatible with rival hardware and the other firm doesn't. The advertising price is highest (lowest) for a compatible (incompatible) firm in an asymmetric compatibility regime than in other regimes. Moreover, distinct from previous work, even with ex-ante symmetric firms, an asymmetric compatibility regime can arise in equilibrium due to preference for variety. From a welfare point of view, we find that mandating compatibility of all applications to all devices is not always socially optimal.

Keywords: Platforms; Compatibility; Preference for Variety; Regulation.

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1 Introduction

A highly competitive and dynamic market for digital products such as virtual reality headsets, tablets, etc., and the associated software are together significant in determining the benefits for consumers and businesses. A crucial strategic choice shaping the competitive dynamics of such digital ecosystems is the compatibility decisions of big tech firms (e.g., Meta, Microsoft, Apple, Amazon) of their application software with rival hardware devices. For instance, Microsoft has made its Xbox Game Pass application compatible with Meta's virtual reality (VR) headsets, so Meta users can access and play Microsoft's exclusive Xbox game titles on Meta VR headset.[1] In contrast, Meta Quest Plus remains incompatible with Microsoft's VR headsets, so users cannot access Meta's VR games on them. A similar asymmetry appears in the tablet market: the Xbox game application is available on Apple iPad, however Apple Arcade is not available on Microsoft tablets. In the smartphone market, both Apple and Huawei have chosen incompatibility: iCloud is available only on the iPhone, and Huawei Mobile Cloud only on Huawei phones. Moreover, Amazon applications (e.g., Amazon Prime) are compatible with Apple devices (e.g., iPhone, iPad) and Apple applications (e.g., Apple TV) are compatible with Amazon devices (e.g., Fire Tablets). Thus, digital markets can exhibit three distinct regimes: one-sided compatibility, mutual incompatibility, and mutual compatibility, depending on firms' strategic incentives.

Previous literature has examined competitive effects of compatibility in digital ecosystems in markets with symmetric hardware qualities (Matutes and Regibeau, 1988), asymmetric hardware qualities (Hurkens et al., 2019, Adner et al., 2020), developer costs (Maruyama and Zennyo, 2015), three symmetric firms (Innocenti and Menicucci, 2021), and product life cycles (Maruyama and Zennyo, 2013). However, a key feature of many digital markets remains unexamined in how it interacts with compability incentives: consumers' preference to accessing a variety of application content. For example, in the online gaming market, each platform offers exclusive game titles, so game users often subscribe to multiple services to access exclusive games available on different platforms. Likewise, in the online video streaming market, consumer preference for variety

[1]See, "Xbox Cloud Gaming app comes to Meta Quest VR headsets, lets you play hundreds of games,"*Indian Express*, available at https://indianexpress.com/article/technology/gaming/xbox-cloud-gaming-app-comes-to-meta-quest-vr-headsets-lets-you-play-hundreds-of-games-9067747/ (accessed on 18-04-2024).

arises because of unique content offered by popular video-streaming service providers.[2] In the cloud storage market, users may maintain multiple cloud storage accounts to cover a variety of needs (e.g., photo backup, document collaboration, large-file sharing).

We intend to fill this research gap and contribute to the theory of compatibility choice by examining the following research questions while accounting for consumers' preference for variety: (1) What are the market conditions for different compatibility regimes to exist? (2) How do market parameters affect firms' compatibility strategies? (3) What is the impact of firms' compatibility decisions on social welfare? To answer these questions, we develop a game-theoretic model with two firms offering differentiated and paid hardware devices and paid software applications (containing exclusive content) to a unit mass of users. The user base is segmented into two groups: a fraction of users are single-product users, consuming only hardware, while the remaining fraction of users are multi-product users, consuming both hardware and applications. All users decide which firm's hardware device to consume. However, crucially, multi-product users have a preference for a variety of application content and can consume all applications available on a hardware device. Additionally, a unit mass of advertisers seeks to reach users and decides whether or not to place advertisements in a firm's hardware device. The timing is sequential. First, firms make decision about the compatibility of applications, followed by setting prices for hardware, applications, and advertisements. Finally, advertisers and consumers make adoption decisions.

Using this tractable model, we derive a set of results. To observe the intuition behind the equilibrium outcomes, note that the impact of application compatibility on the firm's profit is a double-edged sword. In our framework, a preference for a variety of applications leads to two opposite effects for the firm choosing compatibility. On the one hand, its application demand expands. On the other hand, its hardware demand decreases, which in turn decreases advertising revenue and also hardware revenue (if rival is incompatible). Moreover, a firm incurs a cost to make its application compatible with the rival hardware. Our analysis evaluates these trade-offs, yielding novel insights.

First, in contrast to previous studies (e.g. Adner et al., 2020; Maruyama and Zennyo, 2015), we show that an asymmetric compatibility regime (when one firm makes its application compatible

[2] See, for e.g., https://nscreenmedia.com/svod-content-overlap/ (accessed on 01-12-2024).

with rival hardware and the other firm doesn't) leads to the lowest hardware prices relative to full compatibility and incompatibility regimes. Intuitively, since users have a preference for variety, an asymmetric compatibility regime results in vertical product differentiation. This strengthens the incompatible firm's incentive to reduce hardware price to attract more users for its application, and, in turn, the compatible firm also reduces hardware price (facing strong competition). Thus, hardware price competition intensifies compared to other regimes. Moreover, compared to other regimes, asymmetric compatibility results in lowest (respectively, highest) demand from single-product users for the incompatible (respectively, compatible) firm. Since single-product users spend all their usage time on the hardware, this generates a lower (respectively, higher) per-user expected advertising benefit for incompatible (respectively, compatible) firm. Thus, asymmetric compatibility leads to lower (respectively, higher) advertising price for the incompatible (respectively, compatible) firm relative to other regimes. In addition, a firm charges a higher application price when it is compatible compared to the case when it is incompatible with rival hardware. This is because a compatible firm's application demand is inelastic, thus it can charge an application price to extract the entire surplus of a multi-product user from consuming an application.

Next, previous work examining compatibility decisions has shown that, if firms are symmetric, then asymmetric compatibility regime never arises as an equilibrium outcome (e.g., Adner et al., 2020; Innocenti and Menicucci, 2021). In contrast, we show that even with ex-ante symmetric platforms, asymmetric compatibility can arise in the presence of a preference for variety. Firms can exploit the preference for variety effect asymmetrically, with the compatible firm capturing a higher application revenue, and the incompatible firm focusing on vertical product differentiation to increase hardware and advertising revenue. Our result may explain one sided compatibility in the VR market. Microsoft made its Xbox Cloud Gaming application compatible with Meta's Oculus headset to focus on application revenue, whereas Meta's VR games are incompatible with Microsoft's Hololen headset so as to generate higher hardware and advertising revenue. Moreover, the likelihood of asymmetric compatibility is higher in markets with a larger fraction of users consuming both hardware and applications, weaker hardware device differentiation, and lower advertising revenue per user per unit of time.

Finally, motivated by recent policy initiatives aimed to address compatibility issues in digital markets (e.g., Digital Markets Act 2021; ACCESS Act 2021),[3] we conduct a welfare analysis and show that our results are distinct from previous work on platform markets (e.g., Adner et al., 2020; Maruyama and Zennyo, 2015). First, in contrast to Adner et al. (2020), our main finding is that full compatibility (when both firms make their application compatible with rival hardware) is not always desirable. Moreover, unlike Maruyama and Zennyo (2015), we find that full compatibility is socially desirable only when compatibility costs are sufficiently small and the fraction of multi-product users is sufficiently large; otherwise, either asymmetric compatibility or incompatibility (when both firms make their application incompatible with rival hardware) can yield higher social welfare relative to full compatibility. From a policy point of view, we compare market outcomes with socially optimal outcomes and show that a blanket directive of full compatibility is not always desirable and that the nature of the regulatory intervention should depend on market conditions.

The remainder of the paper is structured as follows. Section 2 discusses the relevant literature. Section 3 sets up the game-theoretic model. Section 4 characterizes the equilibrium outcomes, analyzes the impact of market parameters on firms' compatibility strategies, and discusses managerial implications. Section 5 studies multiple extensions to the baseline model. Section 6 examines the welfare implications of firms' compatibility choices and discusses policy implications. Section 7 concludes the paper. All proofs are deferred to the appendices.

2 Contributions to the Literature

Broadly, our paper builds on and contributes to two strands of literature: the first strand examines platform competition and its impact on pricing and design decisions, and the second strand examines the nature of co-opetition between platforms in digital markets.

2.1 Platform Competition

First, our paper is related to the stream of work on platform competition which studies platform pricing strategy (e.g., Armstrong, 2006; Armstrong and Wright, 2007), and advertising decisions

[3]The European Union's Digital Markets Act (DMA) proposal aims to promote fair competition and consumer choice by requiring large digital platforms to ensure interoperability with third-party services. In the United States, the ACCESS Act proposed by Congress seeks to mandate data portability and interoperability among online platforms to reduce switching costs and foster competition. These initiatives reflect a growing recognition among policymakers of the importance of compatibility in shaping the competitive dynamics of digital ecosystems.

(e.g., Anderson and Coate, 2005; Ambrus et al., 2016, Peitz and Valletti, 2008). Our paper deviates from these studies by allowing time allocation between hardware and application(s). This modeling difference is important, as it allows us to obtain new insights on advertising prices. Time allocation crucially leads to a variation in average time spent on hardware/application across the different compatibility regimes. Hence, asymmetric compatibility leads to lower (respectively, higher) advertising price for the incompatible (respectively, compatible) firm relative to other regimes. In addition, unlike the canonical two-sided frameworks of Anderson and Coate (2005) and Armstrong (2006), our model unearths a new cross-subsidization channel: the incompatible firm reduces hardware price not just to attract users for advertisers, but also to attract users who have a stronger preference for variety to its proprietary application.

2.2 Co-opetition Relationship in Digital Markets

Second, our paper contributes to the literature that examines the co-opetition relationship between firms where two or more firms engage in both competition and cooperation in a variety of settings, such as partnership between two complementary product manufacturers on research and development investment while competing on prices (e.g., Casadesus-Masanell and Yoffie, 2007; Niu et al., 2019), and competition between a cluster of competing firms who could benefit from a common complementary product (Yuan et al., 2021).

An important co-opetition relationship in digital markets is platform compatibility decisions. In a system market with multi-product firms, a strand of work has studied firms' compatibility decisions, wherein compatibility allows consumers to mix and match components whereas incompatibility prevents any such combinationsâ€”leading to closed-system competition. The seminal work by Matutes and Regibeau (1988) examine the compatibility incentives of symmetric multi-product firms in a Hotelling setup. They show that incompatibility leads to internalization of price cuts, thus intensifying price competition and yielding lower profits compared to the compatibility regime. A set of papers extend their work and find that incompatibility can arise as an equilibrium, for instance, in the presence of a sufficiently large firm asymmetry in quality (Hurkens et al., 2019) and cost (Hahn and Kim, 2012), and a large number of firms (Kim and Choi, 2015). Our setting differs from these studies in two crucial ways. First, unlike these studies, firms in our setting also earn revenue from advertisements. Second, users in these settings always consume a single component; whereas in our setting, consumers have a preference for variety and can consume both applications available on a hardware. These features generate a new mechanism to explain compatibility decisions: subsidization of hardware to generate higher advertising revenue and also to attract more users for applications (if rival is compatible).

Another strand of work allows for asymmetric compatibility decisions. Liang et al. (2023) study competing platforms of asymmetric scale, where large platform has the ability to host small platforms for users' access. It shows that compatibility is optimal when demand through the compatible channel is in the intermediate range. Closer to our work, Adner et al. (2020) examines competition between platforms that differ in quality, and each must decide whether to make its application compatible with the rival platform. They find that an asymmetric compatibility equilibrium emerges - where the application of one platform is compatible with the rival's device but not vice versa - only if the platforms are ex-ante asymmetric in quality; otherwise, not. In a different setting, Innocenti and Menicucci (2021) derive a similar result and show that, with ex-ante symmetric firms, asymmetric compatibility can never arise as an equilibrium outcome. In contrast to Adner et al. (2020), and Innocenti and Menicucci (2021), we find that even with ex-ante symmetric firms, an asymmetric compatibility regime can arise in equilibrium. Our result hinges on a distinct market characteristic, not examined in these papers: consumers prefer a variety of application content. For intermediate compatibility cost, firms can exploit preference for variety asymmetrically, which intensifies hardware competition, leading to lower hardware prices relative to a full compatibility or incompatibility regime. This enables the incompatible firm to capture higher hardware demand and increase advertising revenue, while the compatible firm captures higher application revenue.

In a related paper, Maruyama and Zennyo (2015) analyze a twoâ€'sided market, where the platform charges both users and application providers and can also choose compatibility at a fixed cost. They show that full compatibility can arise only when the cost of doing so is zero and asymmetric compatibility can arise in markets with small to intermediate compatibility costs. Unlike Maruyama and Zennyo (2015), in our model, a firm obtains revenue from three sources: sales of hardware, application, and advertisements. Moreover, a fraction of users only consume hardware, while the remaining users consume both hardware and application and have a preference for consuming multiple applications. These modeling differences generate new mechanisms and results. First, unlike Maruyama and Zennyo (2015), in our paper, full compatibility can be an equilibrium outcome even at a positive compatibility cost. This is because of a preference for variety, such that the subsidization of hardware to attract users for applications vanishes under full compatibility, leading to higher hardware and application prices compared to other regimes. Thus, higher hardware and application revenue can compensate for the compatibility cost. leading to full compatibility even when the compatibility cost is strictly positive. The preference for variety also leads to new incentives (absent in Maruyama and Zennyo, 2015) to explain asymmetric compatibility. Firms exploit the preference for the variety effect asymmetrically, leading to strong hardware price cutting incentives: the incompatible firm subsidizes hardware to extract higher advertising revenue, whereas the compatible firm shifts the profit focus to application revenue.

Finally, our novel contribution also lies in analyzing the social desirability of a compatibility regime. Both Adner et al. (2020) and Maruyama and Zennyo (2015) find that full compatibility leads to higher social welfare than the full incompatibility and asymmetric compatibility regimes. In contrast, our main finding is that full compatibility is not always desirable. For instance, for sufficiently small compatibility costs and small to intermediate fraction of multi-product users, either asymmetric compatibility or incompatibility can yield higher social welfare relative to full compatibility.

3 Model Preliminaries

3.1 Market Structure

Our model comprises three different types of agents: (i) two firms, 1 and 2 (e.g., Meta and Microsoft), each producing a hardware device H_i, $i \in \{1, 2\}$ (e.g., Oculus VR headset and Hololens VR headset) and an application software A_i, $i \in \{1, 2\}$ (e.g., Meta Quest Plus and Xbox Game Pass), (ii) a unit mass of advertisers, and (iii) a unit mass of users with a fraction $\alpha > 0$ users deriving utility from consuming both hardware device and application(s) available on it (referred to as multi-product users), and the remaining $1 - \alpha$ users deriving utility from consuming only hardware device (referred to as single-product users). Both hardware devices and applications are paid, and in addition, firms obtain revenue from placing advertisements in the hardware.

3.2 Firms' Profit

Each firm $i \in \{1, 2\}$ offers a hardware device H_i and an application A_i. It obtains revenue from three sources: (i) hardware sales at price p_i per user, (ii) application sales at price r_i per user, and (iii) advertisement revenue from placing a_i advertisements on its hardware with price s_i per user charged to an advertiser.[4] Let N_i be the total number of users (both single-product and multi-product) who consume hardware H_i. Among users of hardware H_i, let D_i be the total number of multi-product users who consume hardware H_i, and the remaining $N_i - D_i$ be the total number of single-product users who consume hardware H_i. Let X_i be the total number of users who consume the application A_i. In addition, each firm $i \in \{1, 2\}$ chooses whether or not to make its application A_i compatible with the rival hardware H_j, $j \neq i$, at a fixed compatibility cost $F \geq 0$.[5] If firm i chooses to make its application A_i incompatible with rival hardware H_j, $j \neq i$, then a multi-product user of rival hardware H_j can consume only application A_j, $j \neq i$, available on H_j. Whereas, if firm i chooses to make its application A_i compatible with rival hardware H_j, $j \neq i$, then a multi-product user of rival hardware H_j can consume both applications A_i and A_j available on H_j. Therefore, the profit of firm $i \in \{1, 2\}$ is

$$\pi_i(p_i, r_i, s_i) = p_i N_i + r_i X_i + s_i a_i N_i - \mathbb{I}.F, \tag{1}$$

where $\mathbb{I}$ is the indicator function such that

$$\mathbb{I} = \begin{cases} 0, & \text{if firm } i \text{ chooses incompatibility, and} \\ 1, & \text{if firm } i \text{ chooses compatibility.} \end{cases}$$

3.3 User Utility

There is a unit mass of users. Among them, a fraction $\alpha \in (0, 1)$ users obtain a strictly positive gross utility from consuming both the hardware device and the total number of applications

[4] In Section 5.1, we extend the model to allow advertisements in both hardware and applications. Our main results continue to hold.

[5] The assumption of fixed compatibility cost is in line with previous work (e.g., Maruyama and Zennyo, 2015; Boom, 2001).

available on it. The remaining fraction $1-\alpha > 0$ users only decide which hardware to consume and obtain zero gross utility from the applications. To proceed, we will refer to $\alpha > 0$ users as multi-product users and $1-\alpha > 0$ users as single-product users.

Each user has one unit of time to allocate between using the hardware and application(s). A single-product user spends the entire unit of time on the hardware. However, a multi-product user spends time on both hardware and application(s) available on it. When she consumes one application, she spends a fraction $z \in \left(0, \frac{1}{2}\right)$ unit of time on the application and the remaining $1-z$ unit of time on the hardware; while if two applications are available on the hardware and she consumes both, then she spends z unit of time on each application and the remaining $1-2z$ unit of time on the hardware. A user obtains an intrinsic value $V > 0$ per unit of time spent on a hardware, while the intrinsic value of spending z unit of time on an application is $W(z) > 0$. We assume that $W(z)$ is an increasing and concave function of z, with $W(0) = 0$, $W'(z) > 0$, and $W''(z) < 0$ for all $z \in [0, 1)$.

We assume firms compete à la Hotelling to sell their hardware devices. Hardware devices can be differentiated in the eyes of the users because of their intended use, user interface, etc. For example, Meta and Microsoft virtual reality headsets are differentiated because they provide different virtual experiences. Meta's Oculus allows users to step into fully virtual environments and leave the real world behind, whereas Microsoft's HoloLens allows users to access virtual information without stepping away from their physical environment.[6] Similarly, the Apple iPad is mainly used for work-related activities (e.g. lectures, taking notes), whereas the Amazon Fire tablet is more suitable for entertainment-related purposes (e.g. games, watching movies).[7] User preferences for hardware devices are represented using a Hotelling interval, uniformly distributed over [0,1], with the firm 1′ hardware device located at point 0 and the firm 2′ hardware device located at point 1 of the Hotelling line. A user is characterized by her location x on the Hotelling interval [0,1], where x represents her preference for the ideal product. Each user located at $x \in [0, 1]$ incurs a constant per-unit transportation (misfit) cost t from the consumption of hardware. Thus, she faces a transportation cost of tx (respectively, $t(1-x)$), if she consumes

[6] See, "HoloLens 2 vs Oculus Quest 2: Which is Best?," *XR Today*, available at https://www.xrtoday.com/mixed-reality/hololens-2-vs-oculus-quest-2-which-is-best/ (accessed on 19-04-2024).

[7] See, "Amazon Fire Tablet vs. iPad: Which Is Right for You?," *Lifewire*, available at https://www.lifewire.com/amazon-fire-tablet-vs-ipad-5270471 (accessed on 19-04-2024).

hardware H_1 (respectively, H_2).[8]

Therefore, if a single-product user located at $x \in [0,1]$ purchases hardware H_i at price p_i, then her net utility is

$$U_i(x) = \begin{cases} V - p_1 - tx, & \text{if she consumes } H_1 \ (i=1)\text{, and} \\ V - p_2 - t(1-x), & \text{if she consumes } H_2 \ (i=2). \end{cases} \tag{2}$$

The net utility of a multi-product user located at $x \in [0,1]$ depends on the hardware and the number of applications available on it, which in turn, depends on the compatibility regime. She pays a price p_i for the hardware H_i, and pays an application price r_i (if A_j is not compatible with H_i) or $r_i + r_j$ (if A_j is compatible with H_i). Depending on the compatibility regime, we can have four different scenarios. If application A_j is not compatible with rival hardware H_i, where $j \neq i$, then we are in incompatibility regime NN. A multi-product user's net utility is

$$U_i(x) = \begin{cases} (1-z)V - p_1 + W(z) - r_1 - tx, & \text{if she consumes } H_1 \text{ and } A_1 \ (i=1)\text{, and} \\ (1-z)V - p_2 + W(z) - r_2 - t(1-x), & \text{if she consumes } H_2 \text{ and } A_2 \ (i=2). \end{cases} \tag{3}$$

If application A_j is compatible with rival hardware H_i, where $j \neq i$, then we are in full compatibility regime CC. A multi-product user's net utility is

$$U_i(x) = \begin{cases} (1-2z)V - p_1 + 2W(z) - r_1 - r_2 - tx, & \text{if she consumes } H_1, A_1, \text{ and } A_2 \ (i=1)\text{, and} \\ (1-2z)V - p_2 + 2W(z) - r_2 - r_1 - t(1-x), & \text{if she consumes } H_2, A_1, \text{ and } A_2 \ (i=2). \end{cases} \tag{4}$$

If application A_1 is incompatible with hardware H_2, whereas application A_2 is compatible with hardware H_1, then we are in an asymmetric compatibility regime NC. A multi-product user's

[8] To bring out the main results clearly, we assume that advertisements do not impose a dis-utility on users. A similar assumption was made in previous literature (e.g., Etro 2021, Von Ehrlich and Greiner 2013, Kox et al. 2017). However, in Section 5.4, we extend the baseline model to include nuisance costs of advertisements and find that our main result remains unchanged. For details, please see Online Appendix F.

net utility is

$$U_i(x) = \begin{cases} (1-2z)V - p_1 + 2W(z) - r_1 - r_2 - tx, & \text{if she consumes } H_1, A_1, \text{ and } A_2 \ (i=1), \text{ and} \\ (1-z)V - p_2 + W(z) - r_2 - t(1-x), & \text{if she consumes } H_2 \text{ and } A_2 \ (i=2). \end{cases} \tag{5}$$

If application A_1 is compatible with hardware H_2, whereas application A_2 is incompatible with hardware H_1, then we are in an asymmetric compatibility regime CN. A multi-product user's net utility is

$$U_i(x) = \begin{cases} (1-z)V - p_1 + W(z) - r_1 - tx, & \text{if she consumes } H_1 \text{ and } A_1 \ (i=1), \text{ and} \\ (1-2z)V - p_2 + 2W(z) - r_1 - r_2 - t(1-x), & \text{if she consumes } H_2, A_1 \text{ and } A_2 \ (i=2). \end{cases} \tag{6}$$

3.4 Advertisers' Profit

A unit mass of multi-homing advertisers decides whether or not to place an advertisement in hardware H_i, $i \in \{1,2\}$. An advertiser obtains a revenue (benefit) q per unit of time spent on a hardware per user by placing an advertisement in hardware H_i, and pays a price s_i per user for placing an advertisement in hardware H_i.[9] Let N_i denote the total number of users consuming hardware H_i, wherein D_i (respectively, $N_i - D_i$) are the total number of multi-product (respectively, single-product) users consuming hardware H_i. Recall that a multi-product user spends $1-z$ units of time on hardware H_i if only one application is available on it and $1-2z$ units of time on hardware H_i if both applications are available on it.

If application A_j is not compatible with rival hardware H_i, where $j \neq i$, then we are in an incompatible regime NN. The profit of an advertiser from placing an advertisement in hardware

[9] A similar advertising market (with homogeneous advertisers' valuation for advertisement) is described in Von Ehrlich and Greiner (2013), and Amaldoss et al. (2021). It allows us to clearly describe main economic forces determining equilibrium compatibility regimes. However, in Online Appendix G, we consider a model with heterogeneous advertisers and show that our main results are robust to this extension.

H_i, $i \in \{1,2\}$ is

$$(1-z)qD_i + q(N_i - D_i) - s_i N_i. \tag{7}$$

If application A_j is compatible with rival hardware H_i, where $j \neq i$, then we are in full compatibility regime CC. The profit of an advertiser from placing an advertisement in hardware H_i, $i \in \{1,2\}$ is

$$(1-2z)qD_i + q(N_i - D_i) - s_i N_i. \tag{8}$$

If application A_i is incompatible with hardware H_j, whereas application A_j is compatible with hardware H_i, then we are in an asymmetric compatibility regime. The profit of an advertiser from placing an advertisement in hardware H_i, $i \in \{1,2\}$ is

$$\begin{cases} (1-2z)qD_i + q(N_i - D_i) - s_i N_i, & \text{from placing an ad in hardware } H_i \text{, and} \\ (1-z)qD_j + q(N_j - D_j) - s_j N_j, & \text{from placing an ad in hardware } H_j. \end{cases} \tag{9}$$

3.5 Timeline of the Game

We consider the following five-stage game.

Stage 1: Each firm chooses between compatibility (C) and incompatibility (N). As a result, four market regimes are possible. In the first regime NN, both firms choose incompatibility. In the second regime CC, both firms choose compatibility. In the third regime NC, only firm 2 chooses compatibility, while firm 1 does not, and in the fourth regime CN, only firm 1 chooses compatibility, while firm 2 does not.

Stage 2: Firms 1 and 2 simultaneously and non-cooperatively choose hardware prices p_1 and p_2, application prices r_1 and r_2, and advertising prices, s_1 and s_2, respectively.

Stage 3: Advertisers decide whether or not to advertise in firm i's hardware H_i, $i \in \{1,2\}$.

Stage 4: Users decide whether to purchase hardware H_1 or H_2.

Stage 5: Multi-product users decide whether or not to consume the application(s) available on the hardware device that they have purchased.

The solution concept used is the subgame perfect Nash equilibrium (henceforth equilibrium).

Table 1: Model parameters and variables

Notation	Description
Parameters	
z	Fraction of time spent on an application by a multi-product user.
V	Standalone/intrinsic value obtained per unit of time spent on hardware H_i.
$W(z)$	Standalone/intrinsic value obtained from spending z units of time on application A_i.
t	Per-unit transportation cost.
q	Advertiser revenue (benefit) per unit of time spent on a hardware per user.
F	Cost of making application A_i compatible with rival firm j's hardware H_j.
Variables	
p_i	Price paid by a user to consume firm i's hardware H_i.
r_i	Price paid by a user to consume firm i's application A_i.
s_i	Price paid per-user by an advertiser for placing an advertisement in hardware H_i.
a_i	Level of advertisements in hardware H_i.
N_i	Demand for hardware H_i from all users.
D_i	Demand for hardware H_i from multi-product users.
X_i	Demand for application A_i.

Going forward, to illustrate the intuition behind our analytical results, and also to conduct numerical analysis in some of the later extensions, we shall use the following functional forms:

Example 1. $W(z) = z(1 - b * z) * w$, *where* $w, b, > 0$, *and* $z \in [0, 1]$. *Note that the functional form satisfies all restrictions imposed on* $W(z)$ *for all* $z \in [0, 1]$.

3.6 Discussion of Model Assumptions

3.6.1 Time Allocation

Our model assumes that every user has a fixed total available time (normalized to 1) that is divided between (i) the time spent on the application(s) and (ii) the remaining time spent on the hardware. There is evidence to suggest that users spend time on applications beyond mobile devices. A recent EMarketer report shows that US adults now spend 88% of their mobile internet time inside applications, leaving only 12% for other device usage.[10] Moreover, while the baseline model assumes an exogenously given time allocation between hardware and application, Section 5.3 endogenizes this choice by allowing users to optimally choose their time allocation. Our main results are robust to this extension.

3.6.2 Single-Product and Multi-Product Users

The model assumes some users to be single-product (deriving zero utility from consuming an application), and remaining multi-product users (deriving a positive gross utility from consuming an application). This assumption is in line with empirical observations. For example, a 2022 survey shows that 30% of virtual reality (VR) device consumers used them only for built-in or "core" VR experiences (e.g., watching movies, browsing the internet, virtual workspaces); whereas remaining 70% use them primarily for gaming, which requires downloading or purchasing game titles, and the value of the device increases with the number and variety of those titles. Thus, some consumers are hardware-focused while others obtain value from both hardware and additional applications.[11]

3.6.3 Variety Seeking Behaviour

A key (novel) modeling assumption is that a multi-product user consumes every application available on a hardware as long as they obtain a non-negtaive utility. Therefore, they will purchase and consume all available applications on the hardware. This assumption is in line with empirical observations. For instance, in a 2022 survey, a common consumer complaint while using VR headsets was that there isn't a wide enough selection of games to play on it.[12] An important factor affecting the purchase of multiple applications (e.g., games) is that there exists an incremental value in doing so. This is particularly important in the gaming industry, as both

[10]See, "The Majority of Americans' Mobile Time Spent Takes Place in Apps," *EMarketer*, available at https://www.emarketer.com/content/the-majority-of-americans-mobile-time-spent-takes-place-in-apps?utm_source=chatgpt.com (accessed on 03-05-2025).

[11]See "Beyond Reality 2022," *Group, N.R.*, available at https://assets.ctfassets.net/4ivt4uy3jinr/12b92XBfBiZSYVRBttBLdk/3b47b91d2ba4fa333186f2c3bd69e278/Beyond_Reality_April_2022.pdf (accessed on 19-04-2024).

[12]See,"Beyond Reality 2022," *Group, N.R.*, page 14, available at https://assets.ctfassets.net/4ivt4uy3jinr/12b92XBfBiZSYVRBttBLdk/3b47b91d2ba4fa333186f2c3bd69e278/Beyond_Reality_April_2022.pdf (accessed on 19-04-2024).

Meta and Microsoft own exclusive VR games (e.g., Meta Lone Echo and Microsoft Assasin's Creed). For simplicity, we assume that there is no overlap in the firms' application content.[13]

4 Equilibrium Analysis

We begin by introducing three assumptions that will remain in place throughout the analysis. Assumption 1 below assumes that the intrinsic value obtained consuming an application is large enough to ensure full market coverage for the hardware.

Assumption 1.

$$W(z) \geq zV + 4q$$

Assumption 2 imposes a parametric restriction on the per-unit transportation cost t to ensure non-negative equilibrium prices for both hardware and applications.

Assumption 2.

$$t \geq q + \frac{\alpha W(z)}{3} - \frac{\alpha z V}{3} - \frac{4\alpha z q}{3}$$

We begin by analyzing *Stages 2*, *3*, *4*, and *5* of the game, and derive equilibrium prices, advertisements, and profits for three broad scenarios of competition. (i) Regime NN: incompatibility regime in which neither firm's application is compatible with rival's hardware, (ii) Regime CC: full compatibility regime in which each firm's application is compatible with rival's hardware, and (iii) Regime NC or CN: asymmetric compatibility regime in which firm i's application is not compatible with rival j's hardware, $j \neq i$, whereas rival firm j's application is compatible with firm i's hardware.

[13]Our main results remain unchanged if we allow for partial content overlap among multiple applications. Suppose, for simplicity, the total measure of content/features available on each application is equal to 1 with $\theta \in (0,1]$ units of exclusive content and $1-\theta$ units are non-exclusive/overlapping content. Also, let $W(z)$ be the intrinsic value (utility) obtained from spending z units of time on a unit of content/feature available on application A_i, $i \in \{1,2\}$. Then, if a user consumes application A_1 or A_2, the intrinsic value obtained is $(1-\theta)W(z)+\theta W(z) = W(z)$. However, if a user consumes both applications A_1 and A_2, then the intrinsic value obtained is $W(z) + \theta W(z) = (1+\theta)W(z)$, where $\theta \in (0,1]$, can be interpreted as the incremental benefits from consuming both applications. In this modified framework, our main results will remain unchanged.

4.1 Incompatibility

When neither firm's application is compatible with the rival's hardware, a multi-product user can consume only firm i's own application on its hardware H_i, $i \in \{1, 2\}$. At *Stage 5*, using the individual rationality condition on multi-product users, we obtain the demand X_i for application A_i, $i \in \{1, 2\}$, as

$$X_i = \begin{cases} D_i, & \text{if } r_i \leq W(z) \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

where D_i is the total number of multi-product users who consume hardware H_i. At *Stage 4*, using Equations (2) and (3), (details relegated to Appendix A), we obtain the demand for the hardware devices as

$$\begin{aligned} N_1 &= \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{\alpha(r_2 - r_1)}{2t}, \ N_2 = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha(r_1 - r_2)}{2t}, \\ D_1 &= \alpha\left[\frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{r_2 - r_1}{2t}\right], \text{ and } D_2 = \alpha\left[\frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{r_1 - r_2}{2t}\right]. \end{aligned} \tag{11}$$

At *Stage 3*, advertisers make participation decisions. Given an advertising price s_i, $i \in \{1, 2\}$, an advertiser places an advertisement in hardware H_i, $i \in \{1, 2\}$ if $(1-z)qD_i + q(N_i - D_i) - s_iN_i \geq 0$. This gives the level of advertisements a_i in hardware H_i, $i \in \{1, 2\}$ as

$$a_i = \begin{cases} 1, & \text{if } (1-z)qD_i + q(N_i - D_i) \geq s_iN_i, \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

At *Stage 2*, firm $i \in \{1, 2\}$ chooses hardware price p_i, application price r_i and advertising price s_i to maximize its profit (defined by Equation 1). Let the equilibrium value of a variable y, where y could imply price, advertisement, market share, or profit, be denoted by y^{NN}. The following lemma characterizes the equilibrium.

Lemma 1. *When, at Stage 1, neither firm's application is compatible with rival's hardware, i.e., both* *choose incompatibility, then the equilibrium prices, demands, advertisements, and profits, are as follows:*

$$p_1^{NN} = p_2^{NN} = t - q; \;\; r_1^{NN} = r_2^{NN} = zq; \;\; s_1^{NN} = s_2^{NN} = q - \alpha zq; \;\; N_1^{NN} = N_2^{NN} = \frac{1}{2};$$
$$X_1^{NN} = D_1^{NN} = X_2^{NN} = D_2^{NN} = \frac{\alpha}{2}; \;\; a_1^{NN} = a_2^{NN} = 1; \;\; \textit{and} \;\; \pi_1^{NN} = \pi_2^{NN} = \frac{t}{2}.$$

We highlight three important properties of the preceding equilibrium. First, hardware prices are subsidized for a user by the benefit to the advertiser per unit of time spent on hardware by the user, denoted by q. This is a key property of the two-sided market structure with users and advertising sides, in which, in equilibrium, hardware products are subsidized to compete intensively for users and generate more revenue on the advertising side (see, for e.g., Armstrong and Wright, 2007; Peitz and Valletti, 2008). Thus, the higher q, the lower the price charged for the hardware. As a result, firms' profit are independent of the advertising profit because there is a full pass-through of advertising profit to the users in the form of lower hardware prices. Second, each firm sets application price so as to equate marginal benefit (increased surplus extracted from each multi-product user) against the marginal cost (reduced demand from multi-product users for the hardware and application). Finally, since each multi-product user allocates only a fraction of her unit time to hardware H_i, $i \in \{1, 2\}$, the expected benefit from placing an advertisement in hardware H_i, $i \in \{1, 2\}$ is strictly less than q. Hence, the equilibrium advertising price is less than q.

4.2 Full Compatibility

When both firms choose to make their application compatible with the hardware of the rival firm, a multi-product user can consume the applications of both firms on hardware H_i, $i \in \{1, 2\}$. At *Stage 5*, using the individual rationality condition on multi-product users, we obtain demand X_i for application A_i, $i \in \{1, 2\}$, as

$$X_i = \begin{cases} \alpha, & \text{if } \; r_i \leq W(z) \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

At *Stage 4*, using Equations (2) and (4), (details relegated to Appendix A), we obtain the demand functions for the hardware devices as

$$N_1 = \frac{1}{2} + \frac{p_2 - p_1}{2t}, \; N_2 = \frac{1}{2} + \frac{p_1 - p_2}{2t}, \; D_1 = \alpha\left[\frac{1}{2} + \frac{p_2 - p_1}{2t}\right], \text{ and } D_2 = \alpha\left[\frac{1}{2} + \frac{p_1 - p_2}{2t}\right]. \tag{14}$$

At *Stage 3*, advertisers make participation decisions. Given an advertising price $s_i, i \in \{1,2\}$, an advertiser places an advertisement in hardware $H_i, i \in \{1,2\}$ if $(1-2z)qD_i + q(N_i - D_i) - s_iN_i \geq 0$. This gives the level of advertisements a_i in hardware $H_i, i \in \{1,2\}$ as

$$a_i = \begin{cases} 1, & \text{if } (1-2z)qD_i + q(N_i - D_i) \geq s_iN_i, \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

At *Stage 2*, firm $i \in \{1,2\}$ chooses hardware price p_i, application price r_i and advertising price s_i to maximize its profit (defined by Equation (1)). Let the equilibrium value of a variable y, where y could imply price, advertisement, market share, or profit, be denoted by y^{CC}. The following lemma characterizes the equilibrium.

Lemma 2. *When, at Stage 1, both firms choose to make their application compatible with rival's hardware, i.e., both choose compatibility, then the equilibrium prices, demands, advertisements, and profits, are as follows:*

$$p_1^{CC} = p_2^{CC} = t - q + 2\alpha zq; \; r_1^{CC} = r_2^{CC} = W(z); \; s_1^{CC} = s_2^{CC} = q - 2\alpha zq; \; N_1^{CC} = N_2^{CC} = \frac{1}{2};$$
$$X_1^{CC} = X_2^{CC} = \alpha; \; D_1^{CC} = D_2^{CC} = \frac{\alpha}{2}; \; a_1^{CC} = a_2^{CC} = 1, \; \textit{and} \; \pi_1^{CC} = \pi_2^{CC} = \frac{t}{2} + \alpha W(z) - F.$$

The intuition for equilibrium hardware and advertising prices remains the same as in the incompatible regime. We next turn to application pricing. Using Equation (13), since all multi-product users consume both applications available on the hardware (if they obtain non-negative surplus), the demand for each application is inelastic. As a result, each firm charges an application price so as to extract the entire surplus of a multi-product user from consuming an application.

4.3 Asymmetric Compatibility

Without loss of generality, suppose that firm 2 chooses to make its application A_2 compatible with hardware H_1, whereas firm 1 chooses to make its application A_1 incompatible with hardware H_2. At *Stage 5*, using the individual rationality condition on multi-product users, we obtain the demand X_1 for application A_1, and demand X_2 for application A_2, as

$$X_1 = \begin{cases} D_1, & \text{if } r_1 \leq W(z), \text{ and} \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad X_2 = \begin{cases} \alpha, & \text{if } r_2 \leq W(z), \text{ and} \\ 0, & \text{otherwise,} \end{cases} \tag{16}$$

where D_1 is the total number of multi-product users who consume hardware H_1. At *Stage 4*, using Equations (2) and (5), (details relegated to Appendix A), we obtain the demand functions for the hardware devices as

$$\begin{aligned} N_1 &= \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W(z)}{2t} - \frac{\alpha z V}{2t}, & N_2 &= \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha r_1}{2t} - \frac{\alpha W(z)}{2t} + \frac{\alpha z V}{2t}, \\ D_1 &= \alpha \left[\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{r_1}{2t} + \frac{W(z)}{2t} \right], \text{ and} & D_2 &= \alpha \left[\frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{r_1}{2t} - \frac{W(z)}{2t} \right]. \end{aligned} \tag{17}$$

At *Stage 3*, advertisers make participation decisions. Given an advertising price s_1, an advertiser places an advertisement in hardware H_1, if $(1 - 2z)qD_1 + q(N_1 - D_1) - s_1 N_1 \geq 0$. Similarly, an advertiser places an advertisement in hardware H_2, if $(1 - z)qD_2 + q(N_2 - D_2) - s_2 N_2 \geq 0$. This gives the level of advertisements a_1 in hardware H_1, and a_2 in hardware H_2, as

$$a_1 = \begin{cases} 1, & \text{if } (1 - 2z)qD_1 + q(N_1 - D_1) \geq s_1 N_1, \text{ and} \\ 0, & \text{otherwise,} \end{cases} \quad \text{and } a_2 = \begin{cases} 1, & \text{if } (1 - z)qD_2 + q(N_2 - D_2) \geq s_2 N_2, \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{18}$$

At *Stage 2*, firm $i \in \{1, 2\}$ chooses hardware price p_i, application price r_i and advertising price s_i to maximize its profit (defined by Equation (1)). Let the equilibrium value of a variable y, where y could imply price, advertisement, market share, or profit, be denoted by y^{NC}. The following lemma characterizes the equilibrium.

Lemma 3. *When, at Stage 1, firm 1 chooses to make its application A_1 incompatible with hardware H_2,* *whereas firm 2 chooses to make its application A_2 compatible with hardware H_1, then the equilibrium prices, demands, advertisements, and profits, are as follows:*

$$p_1^{NC} = t - q - \frac{\alpha W(z)}{6} + \frac{\alpha z V}{6} + \frac{2\alpha z q}{3},\ p_2^{NC} = t - q - \frac{\alpha W(z)}{3} + \frac{\alpha z V}{3} + \frac{4\alpha z q}{3};$$

$$r_1^{NC} = \frac{W(z)}{2} - \frac{zV}{2} + zq,\ \ r_2^{NC} = W(z);\quad a_1^{NC} = a_2^{NC} = 1; s_1^{NC} = \frac{q - 2zqD_1^{NC}}{N_1^{NC}},\ s_2^{NC} = \frac{q - 2zqD_2^{NC}}{N_2^{NC}};$$

$$X_1^{NC} = D_1^{NC}, X_2^{NC} = \alpha;\ D_1^{NC} = \alpha\left[\frac{1}{2} + \frac{(3-\alpha)W(z)}{12t} - \frac{(3-\alpha)zV}{12t} - \frac{(3-2\alpha)zq}{6t}\right],\ D_2^{NC} = \alpha\left[1 - D_1^{NC}\right];$$

$$N_1^{NC} = \frac{1}{2} + \frac{\alpha W(z)}{6t} - \frac{\alpha z V}{6t} - \frac{\alpha z q}{6t},\ N_2^{NC} = 1 - N_1^{NC};$$

$$\pi_1^{NC} = (p_1^{NC} + q)N_1^{NC} + (r_1^{NC} - 2zq)D_1^{NC},\ \textit{and}\ \ \pi_2^{NC} = (p_2^{NC} + q)N_2^{NC} + r_2^{NC}(D_1^{NC} + D_2^{NC}) - zqD_2^{NC} - F.$$

Some comments regarding the preceding equilibrium are in place. The hardware prices for a user are subsidized by the benefit to the advertiser per unit of time spent on hardware by the user, denoted by q. This is the same as what was discussed in the previous two regimes. However, unlike the regimes NN and CC, hardware prices are decreasing in α, due to a novel asymmetry in demand elasticity for applications. The demand for the compatible application A_2 is inelastic, which implies that the compatible firm 2 sets r_2 to extract the entire surplus left with users from using the application and does not consider the impact of the hardware price on its application demand. In contrast, the demand for the incompatible application A_1 still depends on its own hardware price p_1 and the application price r_1. Therefore, firm 1 reduces p_1 to attract more multi-product users to increase demand for its application A_1. This unilateral *application-driven* subsidization also forces firm 2 to reduce its hardware price p_2 to attract multi-product users on its device. As a result, both firms end up subsidizing hardware prices to attract multi-product users, which is decreasing in the fraction of the multi-product of users α. Next, the asymmetry in the elasticity of application demand also explains the application prices. Firm 1 sets the price of its application, A_1, balancing the higher surplus extraction effect and the demand reduction effect. However, since application A_2 is compatible with hardware H_1, demand for application A_2 is inelastic, and firm 2 charges application price to extract the entire surplus of a multi-product user for application A_2. Finally, as discussed in Section 4.1, equilibrium advertising price is less than q because a multi-product user spends only a fraction of her unit time on the hardware.

4.4 Comparison of Prices

Having examined *Stages 2, 3, 4* and *5* of the game, we now explain our key equilibrium results. The next proposition characterizes how equilibrium prices for hardware, applications, and advertising differ across the three compatibility regimes. We then turn to *Stage 1* (firms' compatibility decisions) in Proposition 2.

Comparing Lemmas 1, 2, and 3, we obtain the following results.

Proposition 1. *A comparison of equilibrium prices across the three market regimes yields:*

(i) ***Applications prices***: $r_i^{NN} < r_1^{NC} < r_2^{NC} < r_i^{CC}, \quad i \in \{1,2\}$.

(ii) ***Hardware prices***: $p_2^{NC} < p_1^{NC} < p_i^{NN} < p_i^{CC}, \quad i \in \{1,2\}$.

(iii) ***Advertising prices***: $s_1^{NC} < s_i^{CC} < s_i^{NN} < s_2^{NC}, \quad i \in \{1,2\}$.

First, let us consider the application prices. In determining the optimal application pricing strategy, there are two forces at play. First, charging a higher price increases the surplus extracted from each multi-product user. However, it can also reduce the number of multi-product users that consume the firm's hardware and application. The strength of these two effects depends on whether or not a firm's application is compatible with rival hardware. Consider the comparison of application prices under the asymmetric compatibility and incompatibility regimes. Without loss of generality, under an asymmetric compatibility regime, suppose firm 1 chooses incompatibility and firm 2 chooses compatibility. As discussed in Section 4.3, firm 1's application demand is elastic and depends only on its own price, whereas firm 2's application demand is inelastic. Since firm 2 demand is inelastic, it sets the maximum possible application price to extract the entire surplus left with a multi-product user from using the application. However, since the demand for firm 1's application is still elastic, it sets a lower application price (balancing marginal revenue against marginal cost) relative to firm 2, i.e., $r_2^{NC} > r_1^{NC}$. Moreover, the absence of a rival strategic response implies that firm 1 has weaker incentives to reduce its application price to attract multi-product users under the asymmetric compatibility regime NC relative to the incompatibility regime NN. As a result, it sets a higher application price under regime NC relative to regime NN, i.e., $r_1^{NC} > r_i^{NN}$, $i \in \{1,2\}$. Next, under full compatibility regime, application demand is inelastic for both firms, thus both charge maximum possible application price to extract the entire surplus left with a multi-product user from using an application. This implies that $r_i^{CC} = r_2^{NC} > r_1^{NC}$, $i \in \{1,2\}$.

Now, consider hardware prices. The hardware prices are determined by the magnitude of three distinct forces. One, the strength of competition between the firms measured by per-unit transportation cost, t, as is standard under Hotelling price competition. Two, firms compete to attract users for advertisers on their hardware devices because an additional user generates advertising revenue q per unit of hardware time per user. This implies that firms subsidize hardware prices to attract users and make their platforms attractive to advertisers. Three, a firm can further subsidize users to make its hardware device more attractive for the multi-product users, monetizing them through application sales on its hardware. First, consider the comparison of hardware prices under two symmetric regimes: incompatibility and full compatibility regimes. Under the full compatibility regime, the subsidization of hardware to attract multi-product users for application sales vanishes, whereas it is present under the incompatibility regime. This additional force results in lower hardware prices under the incompatibility regime relative to the full compatibility regime, i.e., $p_i^{NN} < p_i^{CC}$, $i \in \{1,2\}$. Next, consider the comparison of hardware prices under an asymmetric compatibility regime with either an incompatibility or a full compatibility regime. Without loss of generality, under an asymmetric compatibility regime, suppose firm 1 chooses incompatibility and firm 2 chooses compatibility. As discussed above, in the asymmetric compatibility regime, the demand for incompatible application A_1 depends only on its own application price r_1. This implies that firm 1 has a stronger incentive to subsidize hardware to attract multi-product users for application A_1 under regime NC relative to regime NN. As a result, firm 1 sets a lower hardware price under regime NC relative to regime NN, i.e., $p_1^{NC} < p^{NN}$. This stronger cross-subsidization by firm 1 under the asymmetric compatibility regime NC also forces compatible firm 2 to lower hardware prices than that under the full compatibility regime, i.e., $p_2^{NC} < p^{CC}$. Finally, since under regime NC, the incompatible firm 1's hardware has more applications available on it than the compatible firm 2's hardware, the former is perceived as a product of higher quality by the multi-product users. This vertical differentiation enables firm 1 to charge a higher hardware price relative to that charged by firm 2, i.e., $p_1^{NC} > p_2^{NC}$.

Our result contrasts with previous work (e.g., Adner et al., 2020; Maruyama and Zennyo, 2015).

Under an asymmetric compatibility regime with independent preferences for hardware and software, Adner et al. (2020) show that incompatibility leads to lowest hardware prices compared to full compatibility and the asymmetric compatibility regime, because under incompatibility, platforms subsidize users by the full amount of content profit. Maruyama and Zennyo (2015) also show that hardware prices are lowest under the incompatibility regime. In contrast to these studies, we show that hardware prices are lowest under an asymmetric compatibility regime. This result stems from a novel aspect of our model, i.e., multi-product users have a preference for variety of application content, which leads to an intense hardware competition between firms. The incompatible firm reduces its hardware price to bring more users for its application, and this in turn makes the compatible firm to reduce its hardware price. Moreover, Maruyama and Zennyo (2015) also show that under full compatibility both platforms charge the lowest (zero) fee to the application provider side due to intense competition for the users. In contrast, we show that application prices increase in the degree of compatibility, so that full compatibility leads to the highest application price rather than the lowest. This reversal stems from the reduction in the elasticity of the application demand with compatibility.

Finally, consider advertising prices. The price that a firm can charge to the advertiser depends on the distribution of single-product and multi-product users, and the time each user spends on hardware. First, consider the comparison of advertising prices under two symmetric regimes: incompatibility and compatibility regimes. A single-product user spends entire one unit of time on the hardware. However, a multi-product user spends $1 - z$ units of time on hardware (when only one application is available) or she spends $1 - 2z$ units of time on hardware (when both applications are available). Therefore, advertisers pay more under the incompatibility regime compared to the compatibility regime, i.e., $s_i^{CC} < s_i^{NN}$, $i \in \{1, 2\}$. Next, consider the comparison of advertising prices under the asymmetric compatibility regimes with the other two symmetric regimes. Comparing Lemmas 1 and 3, we find that the demand from single-product users for compatible firm 2's hardware H_2 is higher under regime NC compared to regime NN. Moreover, a single-product user spends their entire one unit of time on hardware H_2 under both regimes NC and NN. This, in turn, implies that advertisers pay more for placing an advertisement on hardware H_2 under the asymmetric compatibility regime compared to the incompatibility regime, i.e., $s_2^{NC} > s_i^{NN}$, $i \in \{1, 2\}$. Likewise, comparing Lemmas 2 and 3, we find that the demand from single-product users for for incompatible firm 1's hardware H_1 is higher under regime CC compared to regime NC. Thus, advertisers pay more for placing an advertisement in hardware H_1 under full compatibility regime compared to asymmetric compatibility regime, i.e., $s_1^{NC} < s_i^{CC}$, $i \in \{1, 2\}$.

4.5 Comparison of Profits

Now, we analyze *Stage 1*, when firms decide whether or not to choose compatibility. The matrix below summarizes the firms' profit associated with each decision.

		Firm 2	
		Incompatibility	*Compatibility*
Firm 1	*Incompatibility*	$(\pi_1^{NN},\ \pi_2^{NN})$	(π_1^{NC}, π_2^{NC})
	Compatibility	(π_1^{CN}, π_2^{CN})	(π_1^{CC}, π_2^{CC})

Table 2: Payoff matrix

Note that the expression for firms' profit π_i^{NN}, π_i^{CN}, π_i^{NC} and π_i^{CC} are defined in Lemmas 1, 2 and 3. The comparison of profits is summarized in the following proposition and illustrated in Figure 1.

Proposition 2 (Comparison of Profits)**.** *There exist thresholds $F_1(\alpha)$, and $F_2(\alpha)$, with $F_1(\alpha) \geq F_2(\alpha)$, and $\frac{\partial F_1(\alpha)}{\partial \alpha} > \frac{\partial F_2(\alpha)}{\partial \alpha} > 0$, such that in a market characterized by compatibility cost (F) and a fraction of multi-product users (α), the following holds.*

(i) For a sufficiently small compatibility cost, i.e., $F < F_2(\alpha)$, both firms choose compatibility, i.e., regime CC, is the equilibrium outcome. Whereas, for a sufficiently large compatibility cost, i.e., $F \geq F_1(\alpha)$, both firms choose incompatibility, i.e., regime NN, is the equilibrium outcome.

(ii) Otherwise, for an intermediate compatibility cost, i.e., $F_1(\alpha) \leq F < F_2(\alpha)$, there exist multiple equilibria with either firm 1 choosing incompatibility and firm 2 choosing compatibility, i.e., regime NC, or firm 1 choosing compatibility and firm 2 choosing incompatibility, i.e., regime CN, as the equilibrium outcome.

Firm i's decision, $i \in \{1, 2\}$, to make its application compatible with rival hardware depends on the effect of its decision on application revenue, hardware revenue, and advertising revenue. First, the demand for compatible firm i's application increases, a positive effect resulting from the user preference for variety. This raises the ability to charge a higher application price because of inelastic application demand. Thus, application revenue increases with compatibility. Second, the perceived hardware quality of the compatible firm i changes, which in turn depends on the compatibility choice of the rival firm j. If rival firm j chooses incompatibility, then the compatible firm i's hardware is perceived to be lower quality due to fewer applications on it, decreasing demand and price for the firm i's hardware, and thus the hardware revenue decreases. If the rival firm j chooses compatibility, then the perceived quality difference of the compatible firm i's hardware vanishes because both devices have all applications, decreasing hardware demand but increasing the price of the firm i's hardware. On the net, hardware revenue increases. Third, since hardware demand always decreases, irrespective of rival firm j's decision, advertising revenue decreases with compatibility. Finally, if firm i chooses compatibility, then it incurs a compatibility cost F to make application A_i compatible with rival hardware H_j. Therefore, the optimal compatibility decision of firm $i \in \{1, 2\}$ depends on the strength of these trade-offs, which in turn, depends on the levels of compatibility costs (F) and fraction of multi-product users (α).

For a sufficiently small compatibility cost F, i.e., $F < F_2(\alpha)$, if firm i chooses compatibility, then the increase in application revenue dominates the decrease in hardware revenue (if rival is incompatible), decrease in advertising revenue, and compatibility cost. In addition, this holds regardless of the firm's belief about the compatibility decision of the rival firm j, $j \neq i$. Thus, firm i's profit increases with the adoption of compatibility, regardless of its belief about the rival's compatibility decision, i.e. $\pi_i^{CN} > \pi_i^{NN}$, and $\pi_i^{CC} > \pi_i^{NC}$. As a result, each firm's decision to choose compatibility is a dominant strategy, irrespective of the rival's decision, leading to a regime CC in which both firms choose compatibility as the equilibrium outcome. For a sufficiently large compatibility cost F, i.e., $F \geq F_1(\alpha)$, if firm i chooses compatibility, then the increase in application revenue is dominated by the decrease in hardware revenue (if rival is incompatible), the decrease in advertising revenue, and compatibility cost. In addition, this holds regardless of its belief about the compatibility decision of the rival firm j, $j \neq i$. Thus, the firm

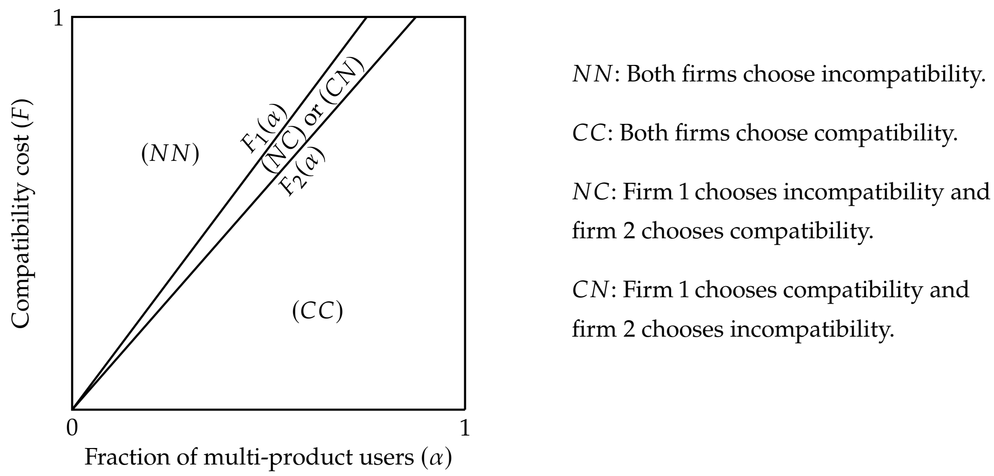

Figure 1: Optimal compatibility regimes under market equilibrium

The figure is based on Example 1 with parameter values $V = 2$, $w = 7$, $q = 0.7$, $t = 1$, $z = 0.3$ and $b = 0.5$. The threshold $F_1(\alpha)$ (respectively, $F_2(\alpha)$) represents the loci of points along which a firm is indifferent between choosing compatibility and incompatibility when it believes that rival has chosen incompatibility (respectively, compatibility).

i's profit decreases with the adoption of compatibility, regardless of its belief about the rival's compatibility decision, i.e., $\pi_i^{CN} < \pi_i^{NN}$, and $\pi_i^{CC} < \pi_i^{NC}$. As a result, each firm's decision to choose incompatibility is a dominant strategy, irrespective of the rival's decision, leading to regime NN with both firms choosing incompatibility as the equilibrium outcome.

Otherwise, for $F_2(\alpha) \leq F < F_1(\alpha)$, the decision of firm i to choose compatibility depends upon its belief about the decision of rival firm j, $j \neq i$. This is because the strength of changes in revenue from choosing compatibility depends upon its belief about the rival's decision. If firm i believes that rival firm j chooses incompatibility, then the increase in application revenue dominates the decrease in hardware and advertising revenues and the compatibility cost. Therefore, if firm i chooses compatibility, its profit increases relative to the case where it chooses incompatibility, i.e.,$\pi_i^{CN} > \pi_i^{NN}$, thus choosing compatibility. If firm i believes that rival firm j chooses compatibility, then the increase in application revenue and hardware revenue is outweighed by the decrease in advertising revenue and compatibility cost. Hence, if firm i chooses compatibility, its profit decreases relative to the case where it chooses incompatibility, i.e., $\pi_i^{CC} < \pi_i^{NC}$, and thus it chooses incompatibility. Therefore, we have multiple equilibria, with either firm 1 choosing incompatibility and firm 2 choosing compatibility, i.e., regime NC, or firm 1 choosing compatibility and firm 2 choosing incompatibility, i.e., regime CN.

As discussed in Section 2, our results differ from previous work. Adner et al. (2020) examine compatibility decisions of vertically differentiated platforms and show that asymmetric compatibility can arise only for sufficiently large differences in hardware qualities; otherwise, either full compatibility or incompatibility is the equilibrium outcome. Moreover, in markets without network effects, Innocenti and Menicucci (2021) show that asymmetric compatibility can never arise as an equilibrium outcome when firms are ex-ante symmetric. In contrast, we show that even with ex-ante symmetric firms, asymmetric compatibility may arise as an equilibrium. In contrast to these studies, we introduce a *new mechanism*, i.e., a preference for variety. This, in turn, generates two distinct effects for a compatible firm: expansion of application demand and vertical differentiation. For intermediate compatibility cost, the two firms exploit preference for variety asymmetrically. Facing an incompatible firm, the compatible firm improves profit by increasing demand for its application; thus, higher application revenue outweighs the decrease in hardware and advertising revenue, and the compatibility cost associated with choosing compatibility. Facing a compatible firm, the incompatible firm improves its profit by creating a vertical hardware differentiation, generating higher hardware demand, and thus higher hardware and advertising revenues.

Maruyama and Zennyo (2015) show that full compatibility arises only if the compatibility cost is zero, and that asymmetric compatibility arises when costs are small to intermediate. In contrast, as discussed above, full compatibility can be sustained even when its cost is strictly positive. This is because multi-product users consume both applications. Therefore, application demand and price increase with compatibility, which offsets the compatibility cost and the decrease in hardware revenue (if rival is incompatible) and advertising revenue. Second, unlike Maruyama and Zennyo (2015), the asymmetric compatibility emerges endogenously through a distinct mechanism: asymmetric exploitation of the preference of variety effect. The incompatible firm subsidizes hardware to increase advertising revenue, while the compatible firm shifts its profit focus toward application sales.

Next, we examine the impact of market parameters, i.e., t and z, on the compatibility strategies of the firms.

Proposition 3 (Comparative Statics). *(i) With an increase in per-unit transportation cost t:* (a)

the region where both firms choose incompatibility, i.e., regime NN, and compatibility, i.e., regime CC is larger, and (b) the region where one firm chooses compatibility and the other firm chooses incompatibility, i.e., regime NC or CN, is smaller.

(ii) With an increase in advertiser revenue per user per-unit of time: (a) the region where both firms choose incompatibility, i.e., regime NN, and compatibility, i.e., regime CC, is smaller and larger, respectively, and (b) the region where one firm chooses compatibility and the other firm chooses incompatibility, i.e., regime NC or CN, is smaller.

The intuition behind the preceding proposition is as follows. Consider the impact of per-unit transportation cost, denoted by t. An increase in t increases the strength of user preferences for hardware devices. As a result, competition between firms to attract users for hardware becomes weaker, thus raising hardware prices. When the rival is incompatible, choosing application compatibility makes the hardware less valuable, and the decrease in hardware revenue with compatibility is steeper. Thus, even for a lower compatibility cost, a firm chooses incompatibility, and the threshold $F_1(\alpha)$ shifts down. When the rival is compatible, choosing application compatibility increases hardware price; however, the decrease in hardware demand with compatibility is weaker (because of stronger hardware preferences). Thus, the increase in hardware revenue with compatibility is greater. As a result, even for a higher compatibility cost, a firm chooses compatibility, and the threshold $F_2(\alpha)$ shifts up. Moreover, since hardware revenue becomes more significant in firms' profit foci, firms would find it less profitable to focus on distinct revenue sources. Thus, the region with an asymmetric compatibility regime becomes smaller.

Consider the impact of an increase in advertiser revenue per user per unit of hardware time, denoted by q. Using Lemma 3, we note that an increase in q decreases (respectively, increases) demand for hardware for incompatible (respectively, compatible) firm under asymmetric compatibility regime. When the rival is incompatible, then choosing application compatibility makes the hardware less valuable; however, the decrease in hardware demand is lower. This, in turn, implies that hardware and advertising revenues decrease by a smaller amount for higher q. Thus, even for a higher compatibility cost, firm chooses compatibility, i.e., thresholds $F_1(\alpha)$ shift up. Similarly, when the rival is compatible, then choosing application compatibility leads to a smaller decrease in hardware demand, thus advertising revenue decreases by a smaller amount. Thus, even for a higher compatibility cost, firm chooses compatibility, i.e., thresholds $F_2(\alpha)$ shift up. Moreover, the region with profit foci of the firms (with incompatible firm focusing on advertising revenue and the compatible firm focusing on application revenue) becomes smaller. This is because a firm has stronger incentives to choose compatibility when facing an compatible firm than when facing an incompatible firm, i.e., $F_2(\alpha)$ increase faster than $F_1(\alpha)$. As a result, the region with an asymmetric compatibility regime becomes smaller.

4.6 Implications for Managers

4.6.1 How should firms price their hardware and applications?

Our results provide recommendations on how a compatible firm should price its application. As our results show, it crucially depends on the rival's compatibility strategy. It should implement a higher application price when faced with a compatible rival firm relative to the case when it faces an incompatible rival firm. This insight is important given that there is an emergence of both full compatibility and asymmetric compatibility regimes in digital markets. Another implication of our result is that, in an asymmetric compatibility regime, firms can reduce their reliance on hardware revenues and exploit different revenue components: the incompatible firm can capture a higher value through advertisements and the compatible firm can capture a higher value through applications. Thus, we should observe a decline in hardware revenue in these regimes. For instance, Meta revenues from virtual reality declined from 2.1 billion USD in 2022 to 1.9 billion U.S. dollars in 2023.[14]

4.6.2 How should firms choose their compatibility strategies?

Based on our results, whether firms should compete or cooperate depends on the opportunity to exploit different revenue streams. This, in turn, depends on how firms can exploit the preference for variety behavior. In markets with sufficiently large or small compatibility cost, the opportunity to capture value through different revenue components is lacking, and firms choose similar strategies to generate either greater hardware revenues or non-hardware revenues. However, with intermediate compatibility costs, firms can exploit consumers' preference for variety

[14]See "Annual revenue generated by Meta Reality Labs segment from 2019 to 2023," *Statista*, available at https://www.statista.com/statistics/1290133/meta-reality-labs-annual-revenue (accessed on 20-04-2024).

asymmetrically, and the opportunity to capture value through different revenue components arises. As argued above, in such markets, an asymmetric compatibility regime arises with the compatible firm focusing on application revenue, whereas the incompatible firm focusing on advertising revenue to improve profit. Moreover, both firms' hardware revenue is lower relative to other regimes. This insight is relevant given that, in many digital markets, technology firms are becoming asymmetrically compatible. Recently, Microsoft made its Xbox Cloud Gaming application compatible with Meta's Oculus headset, whereas Meta's VR games are still incompatible with Microsoft's Hololens headset. This strategy is in line with the revenue focus of the two technology firms, with Meta focusing on improving advertising revenue[15] and Microsoft focusing on improving its application revenue (e.g., from VR games, productivity software) in the VR space.[16]

Another important managerial insight is that as the cost of compatibility decreases, firms' responses can vary depending on the level of compatibility costs. This insight is important given the increased regulatory focus on reducing compatibility costs, for instance, through the adoption of open, standardized, and well-documented APIs.[17] For a sufficiently large compatibility cost, firms' profit foci can change with a decrease in compatibility cost, and both firms are better off in an asymmetric compatibility regime. For intermediate compatibility costs, a decrease in compatibility cost will have symmetric effects on firms' profit foci, and both firms can benefit by choosing compatibility and improving application revenue.

5 Extensions

We next extend the baseline framework in several directions to examine the robustness of our main results.

[15]See "Here's how Zuckerberg thinks Facebook will profit by building a metaverse," *CNBC*, available at https://www.cnbc.com/2021/07/29/facebook-metaverse-plans-to-make-money.html (accessed on 19-04-2024).

[16]See "Analysis of the metaverse strategies of Meta and Microsoft," *Design4real*, available at https://design4real.de/en/the-differences-between-the-metaverse-concept-of-microsoft-and-meta/ (accessed on 20-04-2024).

[17]See, "Building a European Data Economy,"*European Commission*, available at https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=COM:2017:9:FIN (accessed on 22-04-2024).

5.1 Advertisements in Both Hardware and Application

In this section, we allow advertisers to place advertisements in both the hardware device and the application.[18] Let a_{Hi} (respectively, a_{Ai}) denote the number of advertisements in hardware H_i (respectively, application A_i). As shown in Appendix C, on the firm side, each user of application A_i also generates advertising revenue equal to zqa_{Ai}. The rest of the details of the model remain unchanged.

Proposition 4. *When advertisers are allowed to place advertisements in both hardware and application, then our results coincide with the baseline model.*

(i) For a sufficiently small compatibility cost, both firms choose compatibility, i.e., regime CC, is the equilibrium outcome. Whereas, for a sufficiently large compatibility cost, i.e., both firms choose incompatibility, i.e., regime NN, is the equilibrium outcome.

(ii) For an intermediate compatibility cost, the asymmetric compatibility regime NC or CN arises as an equilibrium with one firm choosing incompatibility and the other firm choosing compatibility.

(iii) However, in contrast to the baseline model, the region with full compatibility (incompatibility) regime is larger (smaller).

The main trade-offs characterizing firms' compatibility choices remain similar. When the compatibility cost F is very small, the higher revenue from the application and the higher revenue from the advertisements in the application makes the full compatibility optimal for both firms. When the compatibility cost F is sufficiently large, the gain in hardware revenue from maintaining vertical differentiation dominates the higher revenue from application sales and advertisements in applications, so both firms choose incompatibility. For an intermediate compatibility cost, the incompatible firm focuses on hardware and advertising revenues, whereas the compatible firm focuses on application revenue.

When the rival is incompatible, then, in contrast to the baseline model, making the application

[18] For instance, recently, Amazon prime has allowed for advertisement. See, "Amazon Prime Video in India to get ads starting in June," *TechCrunch*, available at https://techcrunch.com/2025/05/13/amazon-prime-video-in-india-to-get-ads-starting-in-june/ (accessed on 17-05-2025). In the VR market, Meta has started placing advertisements in some of its VR games. See, "Facebook will start putting ads in Oculus Quest apps," *The Verge*, available at https://www.theverge.com/2021/6/16/22535511/facebook-ads-oculus-quest-vr-apps (accessed on 10-03-2025).

compatible is more valuable due to the increase in the revenue from advertisements in the application. Thus, compared to the baseline model, even for a sufficiently high compatibility cost, firms choose compatibility. Similarly, when the rival is compatible, then in contrast to the baseline model, firms gain higher advertising revenue by choosing compatibility as there is no full pass through of advertising revenue (through hardware prices). Thus, compared to the baseline model, even for a high compatibility cost, firm i chooses compatibility.

5.2 Endogenous Hardware Multi-Homing

We now allow a user to purchase one or both hardware devices. Under all three regimes, a single-homing user's utility (consuming a single hardware) remains the same as defined in the baseline model by Equation (2). Suppose a multi-homing single-product user consuming both hardware devices spends λ (respectively, $1-\lambda$) unit of time on hardware H_1 (respectively, hardware H_2). If a single-product user multi-homes and consumes both hardware devices, then her utility is

$$V - p_1 - p_2 - t. \tag{19}$$

If a multi-product user single-homes, then her utility is defined by Equation (3) (under regime NN), Equation (4) (under regime CC), and Equation (5) (under regime NC). A multi-product user who multi-homes always spends z unit of time on each application, and the remaining $(1-2z)$ unit of time on devices H_1 and H_2. Moreover, suppose that a multi-homing multi-product user consuming both hardware devices spends $\lambda(1-2z)$ (respectively, $1-\lambda(1-2z)$) unit of time on hardware H_1 (respectively, hardware H_2). Therefore, if a multi-product user multi-homes and consumes both hardware devices, then her utility is

$$V(1-2z) - p_1 - p_2 + 2W(z) - r_1 - r_2 - t \tag{20}$$

The proposition below characterizes firms' compatibility choices.

Proposition 5. *When users are allowed to multi-home over hardware, that is, they can choose to consume either one hardware device (single-home) or both hardware devices (multi-home), then the following holds:*

(i) For a sufficiently small compatibility cost, both firms choose compatibility, i.e., regime CC, is the equilibrium outcome. Whereas, for a sufficiently large compatibility cost, i.e., both firms choose

incompatibility, i.e., regime NN, is the equilibrium outcome.

(ii) For an intermediate compatibility cost, and when users obtain a small value from consuming an application, asymmetric compatibility regime NC or CN arises as an equilibrium with one firm choosing incompatibility and the other firm choosing compatibility.

(iii) However, for an intermediate compatibility cost, and when users obtain a sufficiently large value from consuming an application, then in contrast to the baseline case, there does not an equilibrium with asymmetric compatibility regime. Either both firms choose incompatibility, i.e., regime NN, or both firms choose compatibility, i.e., regime CC, as the equilibrium outcome.

The intuition for parts (i) and (ii) of Proposition 5 remains the same as described in Section 4. The key intuition for part (iii) arises from the fact that, under regime NN, multi-product users have a greater incentive to consume both hardware devices when they obtain a higher value from consuming an application. Recall that under the asymmetric compatibility regime in the baseline model, the incompatible firm earned higher advertising revenue by maintaining vertical differentiation (because applications are available on its device), whereas the compatible firm focused on higher application revenue. However, multi-homing weakens these forces. When users obtain a sufficiently large value from consuming an application, then a large number of users multi-home under regime NN. Therefore, facing an incompatible firm, if the firm stays incompatible, a large number of multiâ€‘product users multi-home, which leads to a higher application demand relative to the baseline case. This implies that choosing compatibility leads to a lower increase in application demand, and thus it chooses incompatibility. This weakening of incentive leads to the elimination of asymmetric compatibility regime in the presence of multi-homing.

5.3 Endogenous Time Allocation between Hardware and Application

We now endogenize the amount of time that a multiâ€‘product user spends on an application. Let $z \in [0, 1]$ denote the fraction of her time spent on each application, so the time spent on hardware H_i is $(1 - z)$ (respectively, $(1 - 2z)$) when one application is available (respectively, both applications are available). To allow for an option to decide the time allocation between hardware and applications, we introduce *Stage 6*, at which users decide how much time to spend on an application. The rest of the game remains the same as described in the baseline model.

Lemma 4. *At Stage 6, a multiâ€'product user who owns hardware H_i, $i \in \{1, 2\}$ chooses z_u satisfying*

$$W'(z_u) = V. \tag{21}$$

Increasing z increases the utility obtained by consuming the application. On the cost side, utility obtained from the hardware is reduced (because of reduced time spent on the hardware). A multi-product user sets z by equating the marginal benefit and marginal cost, yielding the optimal time spent z_u on an application as defined in Lemma 4. Next, since a multi-product user's optimal z_u remains the same across compatibility regimes, firms' trade off between the revenue components remains qualitatively similar. Hence, as shown in Appendix E, the main insights on firms' compatibility choices are robust to this extension.

5.4 Nuisance Costs

As a robustness check, we extend the baseline model to consider the case when advertising imposes a dis-utility on users. Formally, let $\delta \geq 0$ denote the nuisance cost per advertisement (per unit of viewing time). As described in Section 3.3, a single-product user spends the entire unit of time on the hardware. Therefore, after viewing a_i advertisements on hardware H_i, $i \in \{1, 2\}$, she incurs a nuisance cost of δa_i. A multi-product user spends time on both hardware and application(s) available on it. When she consumes one application on hardware H_i, $i \in \{1, 2\}$, she spends a fraction $z \in (0, 1)$ unit of time on the application and the remaining $1 - z$ unit of time on the hardware; while if two applications are available on the hardware H_i, $i \in \{1, 2\}$, and she consumes both, then she spends z unit of time on each application and the remaining $1 - 2z$ unit of time on the hardware. In the former case, she incurs a nuisance cost of $\delta(1 - z)a_i$, whereas in the latter case, she incurs a nuisance cost of $\delta(1 - 2z)a_i$, after viewing a_i advertisements on hardware H_i, $i \in \{1, 2\}$. The rest of the details of the model remain unchanged. As shown in Appendix F, the economic forces that determine the compatibility strategies of the firms are qualitatively the same, so the equilibrium compatibility regimes are consistent with Proposition 2.

6 Welfare Analysis and Policy Implications

6.1 Welfare Analysis

In this section, we compare social welfare between different market regimes. Social welfare is defined as the sum of the surplus of users, the profit of advertisers and the profit of firms. Since prices are just transfers in the model, it equals the sum of the gross surplus of users from the consumption of hardware devices, the sum of the gross surplus of multi-product users from the consumption of applications, total transportation costs, and the surplus of advertisers. To proceed, we define a socially optimal outcome as the market regime which maximizes social welfare. The following proposition summarizes the main results on the welfare analysis and is illustrated in Figure 2.

Proposition 6 (Comparison of Social Welfare)**.** *There exist thresholds $F_{S2}(\alpha)$ and $F_{S3}(\alpha)$, with $0 \leq F_{S2}(\alpha) \leq F_{S3}(\alpha)$, $\frac{\partial F_{S2}(\alpha)}{\partial \alpha} > 0$, and $\frac{\partial F_{S3}(\alpha)}{\partial \alpha} > 0$, such that in a market characterized by compatibility cost (F) and a fraction of multi-product users (α), the following holds.*

(i) For a sufficiently small compatibility cost, i.e., $0 \leq F < F_{S2}(\alpha)$, full compatibility regime CC is socially optimal. Whereas, for a sufficiently large compatibility cost, i.e., $F \geq F_{S3}(\alpha)$, incompatibility regime NN is socially optimal.

(ii) Otherwise, for an intermediate compatibility cost, i.e., $F_{S2}(\alpha) \leq F < F_{S3}(\alpha)$, asymmetric compatibility regimes NC and CN are socially optimal.

In order to explain the intuition behind the preceding proposition, four important observations must be highlighted. First, the sum of the gross surplus of multi-product users from applications is the largest under the full compatibility regime CC. Second, total compatibility costs are also the largest under the full compatibility regime CC. Third, total transportation costs are lower under the symmetric full compatibility regime CC and the symmetric incompatibility regime NN relative to the asymmetric compatibility regimes NC and CN. Fourth, advertising surplus is the largest under incompatibility regime NN relative to other regimes. Therefore, for a sufficiently small compatibility cost, i.e., $0 \leq F < F_{S2}(\alpha)$, the full compatibility regime maximizes multi-product users' gross surplus from applications, and together with weakly lower transportation costs relative to other regimes, maximizes social welfare. However, for a sufficiently large

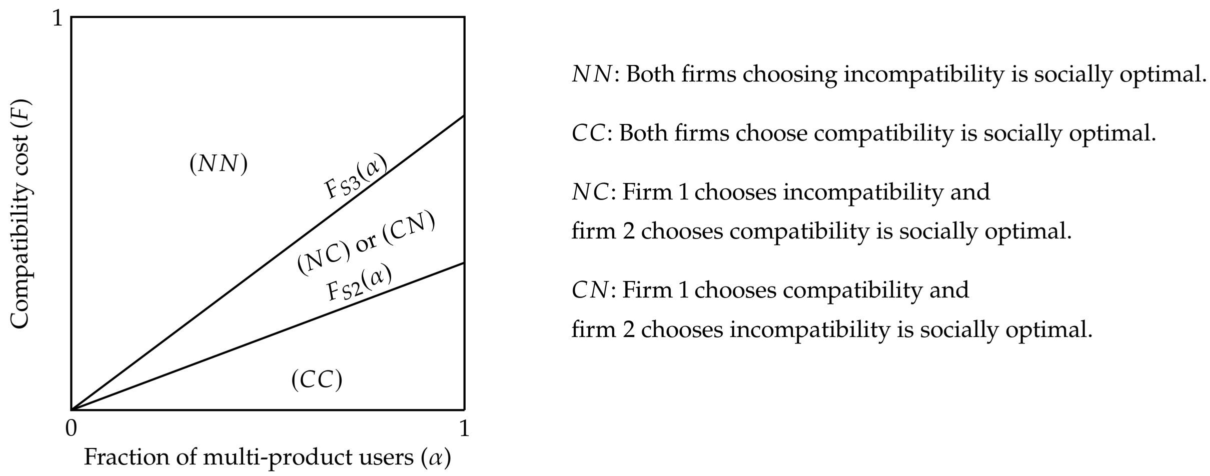

Figure 2: Socially optimal compatibility regimes

The figure is based on Example 1 with parameter values $V = 2$, $w = 7$, $q = 0.7$, $t = 1$, $z = 0.3$ and $b = 0.5$. The threshold $F_{S2}(\alpha)$ (respectively, $F_{S3}(\alpha)$) represents the loci of points along which regime CC (respectively, regime NN) and regime NC or CN yield the same value of social welfare.

compatibility cost, i.e., $F \geq F_{S3}(\alpha)$, the incompatibility regime maximizes advertisers' surplus and together with weakly lower transportation costs relative to other regimes, maximizes social welfare. For an intermediate compatibility cost, that is $F_{S2}(\alpha) \leq F < F_{S3}(\alpha)$, social welfare under the full compatibility regime CC is still lower relative to the incompatibility regime NN due to the high total compatibility costs and the lower advertising surplus in the former case relative to the latter. However, a higher gross surplus of multi-product users from applications under asymmetric compatibility regimes NC and CN offsets higher total transportation costs and compatibility costs, and lower advertisers' surplus, thus increasing social welfare relative to the incompatibility regime NN. Hence, for $F_{S2}(\alpha) \leq F < F_{S3}(\alpha)$, asymmetric compatibility regime NC or CN maximizes social welfare.

Our results on socially optimal compatible choices contrast with the existing work on platform compatibility (e.g., Adner et al., 2020; Maruyama and Zennyo, 2015). First, unlike Adner et al. (2020), we show that full compatibility is not always desirable. An asymmetric compatibility regime can be socially optimal for small to intermediate compatibility costs. Moreover, unlike Maruyama and Zennyo (2015), we find that even for sufficiently small compatibility costs, an asymmetric compatibility regime can be socially optimal for a small to intermediate fraction of multi-product users.

6.2 Comparison of Market and Socially Optimal Outcome

We now compare the market and socially optimal outcomes and examine whether the strategic compatibility decisions of firms are socially optimal. Due to analytical intractability, we conduct a graphical analysis (illustrated in Figures 3) and draw important conclusions.

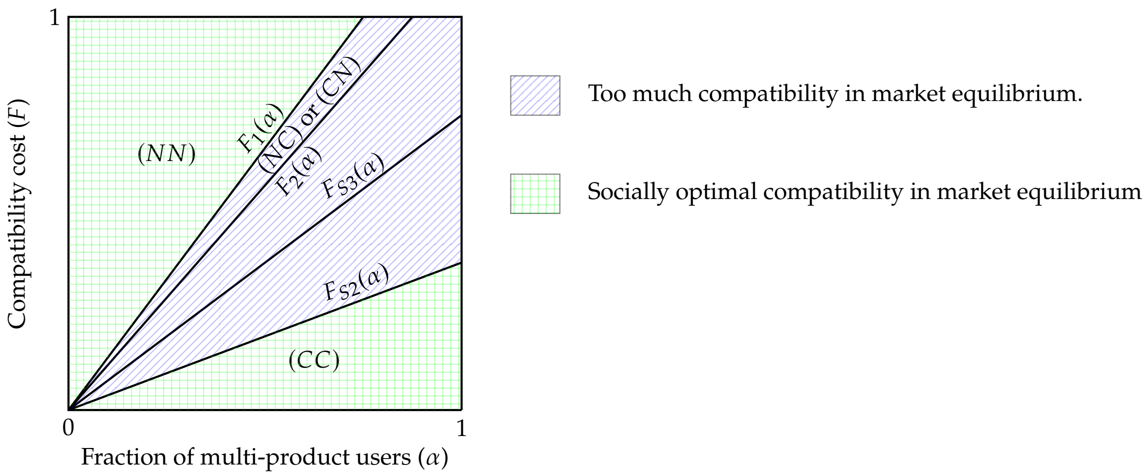

Figure 3: Comparison of optimal compatible regimes under market equilibrium and social optimum

The figure is based on Example 1 with parameter values $V = 2$, $w = 7$, $q = 0.7$, $t = 1$, $z = 0.3$ and $b = 0.5$. The thresholds $F_1(\alpha)$ and $F_2(\alpha)$ (respectively, $F_{S2}(\alpha)$ and $F_{S3}(\alpha)$) are as defined in Proposition 2 (respectively, Proposition 6).

First, in markets with sufficiently large or small compatibility costs, the market outcome coincides with social optimum. Second, for small to intermediate compatibility costs, there can be a mis-alignment of incentives of firms and policymakers (maximizing social welfare) regarding the optimal compatibility regimes. Based on graphical analysis, we find that there can be socially excessive compatibility. However, the nature of comparison depends on the level of compatibility costs. First, consider $F_{S2}(\alpha) \leq F < F_2(\alpha)$. Intuitively, firms' profits are affected symmetrically by compatibility decisions. Thus, both firms choose compatibility to increase application revenue, i.e., regime CC is the equilibrium outcome. However, for the same range of parameter values, from a social point of view, either the asymmetric compatibility regime NC or CN (for $F_{S2}(\alpha) \leq F < F_{S3}(\alpha)$) or the incompatibility regime NN (for $F_{S3}(\alpha) \leq F < F_2(\alpha)$) maximizes social welfare (refer Figure 3). Next, consider $F_2(\alpha) \leq F < F_1(\alpha)$. Firms focus on different revenue components; thus, asymmetric compatibility regime with one firm choosing compatibility and the other firm choosing incompatibility, i.e., regime NC or CN, is the equilibrium outcome. However, for the same range of parameter values, from a social point of view, compatibility costs are sufficiently large, and regime NN maximizes social welfare.

6.3 Policy Implications

6.3.1 Is full compatibility always desirable?

Recent policy debates and exemplified by regulatory initiatives such as the EU Digital Markets Act, have focused on mandatory full compatibility to improve efficiency. Our results caution against a blanket approach and highlight the key forces that a regulator should consider while considering the efficiency of each regime. Based on our analysis, three important trade-offs arise with compatibility: increase in application surplus, decrease in advertising surplus, and compatibility cost. When the cost of making an application compatible is sufficiently small, the regulator should give a higher weightage to the increased application surplus associated with compatibility; thus, full compatibility is socially optimal (which is also the market outcome). Whereas, when the cost of making an application compatible is sufficiently large, the regulator should give a higher weightage to decrease in advertising surplus associated with compatibility, and we find that incompatibility is socially optimal (which is also the market outcome). Since in these markets, firms' evaluation of trade-offs aligns with regulator, market outcome is efficient, and a regulatory intervention may not be required.

6.3.2 In which markets, regulatory intervention is required?

Based on our analysis, regulatory intervention is required in markets with intermediate compatibility costs. However, our analysis shows that a stringent full compatibility regulation that requires all firms' applications to be compatible with their rival devices can be socially suboptimal. Therefore, the regulator needs to carefully examine the trade-offs and accordingly adopt compatibility policies depending on observable market parameters (e.g., fraction of multi-product users, degree of hardware differentiation). In such markets, a regulator can better achieve regulatory objectives by implementing either incompatibility or asymmetric compatibility.

7 Conclusion

Co-opetition, a phenomenon involving simultaneous competition and cooperation, is becoming more prevalent among firms in digital markets. A key facet of co-opetitive behavior is when a digital platform makes its application compatible with the rival firm's hardware device. We examine the issue of application compatibility using a theoretical model with two firms, each having a hardware device and an application, a set of users with a fraction of single-product users, and the remaining multi-product users, and a set of advertisers. We examine equilibrium market outcomes, their properties, and socially optimal compatibility choices. We find several interesting insights. Prices of hardware, applications, and advertisements differ across regimes: for example, application prices are highest under full compatibility and lowest under incompatibility, whereas hardware prices are lowest under asymmetric compatibility. When compatibility costs are small, full compatibility arises as an equilibrium; whereas when costs are large, full incompatibility arises. For intermediate compatibility costs, asymmetric compatibility emerges endogenously – even with ex-ante symmetric firms – because the compatible firm exploits preference for variety to increase application revenue while the incompatible firm differentiates its hardware to increase advertising revenue. From a welfare perspective, full compatibility is socially optimal only when costs are very small; for the intermediate to large compatibility cost, asymmetric compatibility or incompatibility maximizes social welfare.

These results yield an important managerial implication. Based on the trade-offs examined, while making compatibility decisions, firms should consider not only the direct cost of making applications compatible with rival hardware but also the change in revenue from hardware, application, and advertisements. Our analysis also highlights an important policy pitfall. Regulators should avoid the one-size-fits-all mandate of full compatibility. In particular, in markets with intermediate compatibility costs, asymmetric compatibility requirements (for instance, imposing APIâ€‘access obligations only on dominant platforms) may better align private incentives with social welfare.

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Online Appendices for "Application Compatibility in the Presence of Preference for Variety"

This online appendix contains proofs of the lemmas and propositions in Sections 4, 5, and 6, and the proofs behind some of the results and claims in the main text.

A Derivation of Demand Functions

In this section, we derive the demand functions for both the hardware and the application of each firm, in each of the possible market regimes: incompatibility regime NN, full compatibility regime CC, and asymmetric compatibility regime NC or CN.

Demand from single-product users $(1 - \alpha > 0)$

In all regimes, a fraction $1 - \alpha > 0$ of single-product users derive utility from consuming only hardware $H_i,\ i \in \{1, 2\}$ and no utility from consuming application $A_i,\ i \in \{1, 2\}$, i.e., single-product users compare the benefits and costs only of H_1 and H_2 to make their decisions. Using Equation (2), the single-product user indifferent between hardware H_1 and H_2 is located at $\hat{x}$, such that (i) $U_1(\hat{x}) = U_2(\hat{x})$, (ii) $U_1(x) > U_2(x)$, for all $x < \hat{x}$, and (iii) $U_1(x) < U_2(x)$, for all $x > \hat{x}$, where

$$\hat{x} \ = \ \frac{1}{2} + \frac{p_2 - p_1}{2t}. \tag{A.1}$$

Demand from multi-product users $(\alpha > 0)$

Incompatibility (NN)

When each firm $i \in \{1, 2\}$ chooses to make its applications A_i incompatible with the rival firmâ€™s hardware $H_j, j \neq i$, then a fraction $\alpha > 0$ of multi-product users can only consume application A_i available on firm i's hardware H_i. Using Equation (3), the multi-product user indifferent between hardware H_1 and H_2 is located at $\bar{x}$, such that (i) $U_1(\bar{x}) = U_2(\bar{x})$, (ii) $U_1(x) > U_2(x)$, for all $x < \bar{x}$, and (iii) $U_1(x) < U_2(x)$, for all $x > \bar{x}$, where

$$\bar{x} \ = \ \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{r_2 - r_1}{2t}. \tag{A.2}$$

Using Equation (A.2), demand for hardware H_1 and H_2 from multi-product users is

$$\begin{aligned} D_1 &= \alpha\bar{x} = \alpha\left[\frac{1}{2}+\frac{p_2-p_1}{2t}+\frac{r_2-r_1}{2t}\right], \text{ and} \\ D_2 &= \alpha(1-\bar{x}) = \alpha\left[\frac{1}{2}+\frac{p_1-p_2}{2t}+\frac{r_1-r_2}{2t}\right]. \end{aligned} \tag{A.3}$$

Combining Equations (A.1) and (A.2) gives the total demand for hardware H_1 and H_2 (from both single-product and multi-product users) as

$$\begin{aligned} N_1 &= (1-\alpha)\hat{x}+\alpha\bar{x} = \frac{1}{2}+\frac{p_2-p_1}{2t}+\frac{\alpha(r_2-r_1)}{2t}, \text{ and} \\ N_2 = 1-N_1 &= \frac{1}{2}+\frac{p_1-p_2}{2t}+\frac{\alpha(r_1-r_2)}{2t}. \end{aligned} \tag{A.4}$$

Full compatibility (CC)

When each firm $i \in \{1,2\}$ chooses to make its applications A_i compatible with the rival firmâ€™s hardware H_j, $j \neq i$, then a fraction $\alpha > 0$ of multi-product users can consume both applications A_i and A_j on firm i's hardware H_i. Using Equation (4), the multi-product user indifferent between hardware H_1 and H_2 is located at $\bar{x}$, such that (i) $U_1(\bar{x}) = U_2(\bar{x})$, (ii) $U_1(x) > U_2(x)$, for all $x < \bar{x}$, and (iii) $U_1(x) < U_2(x)$, for all $x > \bar{x}$, where

$$\bar{x} = \frac{1}{2}+\frac{p_2-p_1}{2t}. \tag{A.5}$$

Using Equation (A.5), demand for hardware H_1 and H_2 from multi-product users is

$$\begin{aligned} D_1 &= \alpha\bar{x} = \alpha\left[\frac{1}{2}+\frac{p_2-p_1}{2t}\right], \text{ and} \\ D_2 &= \alpha(1-\bar{x}) = \alpha\left[\frac{1}{2}+\frac{p_1-p_2}{2t}\right]. \end{aligned} \tag{A.6}$$

Combining Equations (A.1) and (A.5) gives the total demand for hardware H_1 and H_2 (from both single-product and multi-product users) as

$$\begin{aligned} N_1 &= (1-\alpha)\hat{x}+\alpha\bar{x} = \frac{1}{2}+\frac{p_2-p_1}{2t}, \text{ and} \\ N_2 &= 1-N_1 = \frac{1}{2}+\frac{p_1-p_2}{2t}. \end{aligned} \tag{A.7}$$

Asymmetric compatibility (NC or CN)

Without loss of generality, suppose that firm 1 chooses to make its application A_1 incompatible with rival firmâ€™s hardware H_2, whereas firm 2 chooses to make its application A_2 compatible with rival firmâ€™s hardware H_1. This implies that a multi-product user can consume both applications A_1 and A_2 on hardware H_1, but can only consume application A_2 on hardware H_2. Using Equation (5), the multi-product user indifferent between hardware H_1 and H_2 is located at $\bar{x}$, such that (i) $U_1(\bar{x}) = U_2(\bar{x})$, (ii) $U_1(x) > U_2(x)$, for all $x < \bar{x}$, and (iii) $U_1(x) < U_2(x)$, for all $x > \bar{x}$, where

$$\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{r_1}{2t} + \frac{W(z)}{2t} - \frac{zV}{2t}. \tag{A.8}$$

Using Equation (A.8), demand for hardware H_1 and H_2 from multi-product users is

$$\begin{aligned} D_1 &= \alpha\bar{x} = \alpha\left[\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{r_1}{2t} + \frac{W(z)}{2t} - \frac{zV}{2t}\right], \text{ and} \\ D_2 &= \alpha(1-\bar{x}) = \alpha\left[\frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{r_1}{2t} - \frac{W(z)}{2t} + \frac{zV}{2t}\right]. \end{aligned} \tag{A.9}$$

Combining Equations (A.1) and (A.8) gives the total demand for hardware H_1 and H_2 (from both single-product and multi-product users) as

$$\begin{aligned} N_1 &= (1-\alpha)\hat{x} + \alpha\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W(z)}{2t} - \frac{\alpha zV}{2t}, \text{ and} \\ N_2 &= 1 - N_1 = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha r_1}{2t} - \frac{\alpha W(z)}{2t} + \frac{\alpha zV}{2t}. \end{aligned} \tag{A.10}$$

This completes the derivation of demand functions for hardware and applications.

B Proofs of the Technical Results

Proof of Lemma 1

At *Stage 3*, note that an advertiser decides to place an advertisement in firm i's hardware H_i, $i \in \{1, 2\}$, if he obtains a non-negative profit. Using the advertising demand function (defined by Equation (12)), we have

$$s_i = \frac{(1-z)qD_i + q(N_i - D_i)}{N_i}, \text{ and } a_i = 1, \; i \in \{1, 2\}. \tag{B.1}$$

Any s_i above this level would yield a negative payoff to advertiser (so $a_i = 0$), and any s_i below it would yield $a_i = 1$ but a lower revenue to firm i. Hence, at *Stage 2* in equilibrium s_i and a_i are as defined by Equation (B.1).

Second, since a multi-product user's reservation utility from the app is $W(z)$, any application price $r_i > W(z)$, yields zero application demand; hence an optimal r_i must be between $[0, W(z)]$. Therefore, using the application demand function (defined by Equation (10)), we have $X_i = D_i$. Now, using s_i and a_i (defined by Equation (B.2)) in Equation (1), the profit of firm $i \in \{1, 2\}$ is

$$\pi_i = (p_i + q)N_i + (r_i - zq)D_i.$$

Substituting demands from Equations (A.3) and (A.4) in the preceding profit functions, the first-order conditions to firm i's profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{2} + \frac{p_j - p_i}{2t} + \alpha \frac{(r_j - r_i)}{2t} + (p_i + q)\left(\frac{-1}{2t}\right) + (r_i - zq)\left(\frac{-\alpha}{2t}\right) \leq 0, \text{ and} \tag{B.2}$$

$$\frac{\partial \pi_i}{\partial r_i} = (p_i + q)\left(\frac{-\alpha}{2t}\right) + \frac{\alpha}{2} + \alpha \frac{(p_j - p_i)}{2t} + \alpha \frac{r_j - r_i}{2t} + (r_i - zq)\left(\frac{-\alpha}{2t}\right) \leq 0, \tag{B.3}$$

where subscript $j \neq i$ denotes the rival firm. Using Equations (B.2) and (B.3), setting $\frac{\partial \pi_i}{\partial p_i} = 0$, and $\frac{\partial \pi_i}{\partial r_i} = 0$, and imposing symmetry (since firms have symmetric objective functions), we obtain the symmetric optimal hardware prices as

$$p_1^{NN} = p_2^{NN} = t - q, \tag{B.4}$$

which are non-negative, given the assumption of non-negative prices (Assumption 2), and symmetric optimal application prices are

$$r_1^{NN} = r_2^{NN} = zq. \tag{B.5}$$

Since each firmâ€™s profit function π_i is strictly concave in its own choice variables: p_i and r_i (holding the rivalâ€™s prices fixed), and also, the symmetric solution to the firstâ€'order conditions is strictly interior (i.e., positive), given the assumption of non-negative prices (Assumption 2)), the interior solution constitutes the unique global maximum.

The equilibrium demand for hardware H_i is $N_i = \frac{1}{2}, i \in \{1, 2\}$, the equilibrium demand for application A_i is $X_i = D_i = \frac{\alpha}{2}, \ i \in \{1, 2\}$ (because $W(z) > r^{NN} = zq$), and the equilibrium advertising level is $a_1^{NN} = a_2^{NN} = 1$. Combining this with Equation (B.1), we have

$$s_1^{NN} = s_2^{NN} = q - \alpha z q. \tag{B.6}$$

Using Equation (1), firms' profit are $\pi_1^{NN} = \pi_2^{NN} = \frac{t}{2}$. This completes the proof. ■

Proof of Lemma 2

At *Stage 3*, note that an advertiser decides to place an advertisement in firm i's hardware H_i, $i \in \{1, 2\}$, if he obtains a non-negative profit. Using the advertising demand function (defined by Equations (15)), we have

$$s_i = \frac{(1-2z)qD_i + q(N_i - D_i)}{N_i}, \quad \text{and} \quad a_i = 1, \ i \in \{1, 2\}. \tag{B.7}$$

As discussed in the proof of Lemma 1, any s_i above this level would yield a negative payoff to advertiser (so $a_i = 0$), and any s_i below it would yield $a_i = 1$ but a lower revenue to firm i. Hence, at *Stage 2* in equilibrium s_i and a_i are as defined by Equation (B.7).

Second, since a multi-product user's reservation utility from the application is $W(z)$, any application price $r_i > W(z)$, yields zero application demand; hence an optimal r_i must be between $[0, W(z)]$. Therefore, using the application demand function (defined by Equation (13)), we have $X_i = D_i + D_j = \alpha$. Now, using these values in Equation (1), the profit of firm $i \in \{1, 2\}$ is

$$\pi_i = (p_i + q)N_i + r_i\alpha - 2zqD_i - F.$$

Substituting demands from Equations (A.6) and (A.7) in the preceding profit functions, the

first-order conditions to firm i's profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{2} + \frac{p_j - p_i}{2t} + (p_i + q)\left(\frac{-1}{2t}\right) - 2zq\alpha\left(\frac{-1}{2t}\right) \leq 0, \text{ and} \tag{B.8}$$

$$\frac{\partial \pi_i}{\partial r_i} = \alpha > 0, \tag{B.9}$$

where subscript $j \neq i$ denotes the rival firm. Using Equations (B.8) and (B.9), setting $\frac{\partial \pi_i}{\partial p_i} = 0$, and imposing symmetry (since firms have symmetric objective functions), we obtain the symmetric optimal hardware prices as

$$p_1^{CC} = p_2^{CC} = t - q + 2\alpha zq, \tag{B.10}$$

which are non-negative, given the assumption of non-negative prices (Assumption 2). Since each firmâ€™s profit function π_i is strictly concave in its own choice variable: p_i (holding the rivalâ€™s price fixed), and also, the symmetric solution to the firstâ€'order conditions is strictly interior (i.e., positive), given the assumption of non-negative prices (Assumption 2), the interior solution constitutes the unique global maximum.

Equation (B.9) shows that the profits are strictly increasing in application prices. Thus, firm $i \in \{1, 2\}$ sets r_i to extract as much consumer surplus as possible, while also ensuring that all multi-product users consume application A_i, i.e., users obtain a non-negative payoff from consuming application A_i. Using the application demand function (defined by Equation (13)), we have

$$r_1^{CC} = r_2^{CC} = W(z). \tag{B.11}$$

The equilibrium demand for hardware H_i is $N_i = \frac{1}{2}, i \in \{1, 2\}$, the equilibrium demand for application A_i is $X_i = D_i = \alpha,\ i \in \{1, 2\}$, and the equilibrium advertising level is $a_1^{NN} = a_2^{NN} = 1$. Combining this with Equation (B.7), we have

$$s_1^{CC} = s_2^{CC} = q - 2\alpha zq. \tag{B.12}$$

Using Equation (1), firms' profit are $\pi_1^{CC} = \pi_2^{CC} = \frac{t}{2} + \alpha W(z) - F$. This completes the proof. ■

Proof of Lemma 3

Without loss of generality, suppose that firm 1 chooses to make its application A_1 incompatible with rival firmâ€™s hardware H_2, whereas firm 2 chooses to make its application A_2 compatible with rival firmâ€™s hardware H_1. At *Stage 3*, note that an advertiser decides to place an advertisement in firm i's hardware H_i, $i \in \{1,2\}$, if he obtains a non-negative profit. Using the advertising demand function (defined by Equation (18)), we have

$$s_1 = \frac{(1-2z)qD_1 + q(N_1 - D_1)}{N_1}, \; s_2 = \frac{(1-z)qD_2 + q(N_2 - D_2)}{N_2}, \; \textit{and} \; a_1 = a_2 = 1. \tag{B.13}$$

As discussed in the proof of Lemma 1, any s_i above this level would yield a negative payoff to advertiser (so $a_i = 0$), and any s_i below it would yield $a_i = 1$ but a lower revenue to firm i. Hence, at *Stage 2* in equilibrium s_i and a_i are as defined by Equation (B.13).

Second, since a multi-product user's reservation utility from the application is $W(z)$, any application price $r_i > W(z)$, yields zero application demand; hence an optimal r_i must be between $[0, W(z)]$. Therefore, using the application demand function (defined by Equation (16)), we have $X_1 = D_1$ and $X_2 = D_1 + D_2 = \alpha$. Now, using these values in Equation (1), the profit of firm $i \in \{1,2\}$ is

$$\pi_1 = (p_1 + q)N_1 + (r_1 - 2zq)D_1, \text{ and } \pi_2 = (p_2 + q)N_2 + r_2\alpha - zqD_2 - F.$$

Substituting demand from Equations (A.9) and (A.10) in the preceding profit functions, the first-order conditions to firm 1 and firm 2's profit maximization are

$$\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W(z)}{2t} - \frac{\alpha z V}{2t} + (p_1 + q)\left(\frac{-1}{2t}\right) + (r_1 - 2zq)\left(\frac{-\alpha}{2t}\right) \le 0, \tag{B.14}$$

$$\frac{\partial \pi_1}{\partial r_1} = (p_1 + q)\left(\frac{-\alpha}{2t}\right) + \frac{\alpha}{2} + \alpha\frac{(p_2 - p_1)}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W(z)}{2t} - \frac{\alpha z V}{2t} + (r_1 - 2zq)\left(\frac{-\alpha}{2t}\right) \le 0, \tag{B.15}$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha r_1}{2t} - \frac{\alpha W(z)}{2t} + \frac{\alpha z V}{2t} + (p_2 + q)\left(\frac{-1}{2t}\right) + 0 - zq\left(\frac{-\alpha}{2t}\right) \le 0, \text{ and} \tag{B.16}$$

$$\frac{\partial \pi_2}{\partial r_2} = \alpha > 0. \tag{B.17}$$

Using first-order conditions defined by Equations (B.14), (B.15) and (B.16), setting $\frac{\partial \pi_1}{\partial p_1} = 0$, $\frac{\partial \pi_1}{\partial r_1} = 0$, and $\frac{\partial \pi_2}{\partial p_2} = 0$, and solving, we obtain the optimal price of hardware H_1 as

$$p_1^{NC} = t - q - \frac{\alpha W(z)}{6} + \frac{\alpha z V}{6} + \frac{4\alpha z q}{6}, \tag{B.18}$$

and the optimal price of hardware H_2 as

$$p_2^{NC} = t - q - \frac{\alpha W(z)}{3} + \frac{\alpha z V}{3} + \frac{4\alpha z q}{3}, \tag{B.19}$$

which are non-negative, given the assumption of full market coverage (Assumption 1) and non-negative prices (Assumption 2). Similarly, we obtain he optimal price of application A_1 as

$$r_1^{NC} = \frac{W(z)}{2} - \frac{zV}{2} + zq. \tag{B.20}$$

Since firm 1's (respectively, firm 2's) profit function π_1 (respectively, π_2) is strictly concave in its own choice variables: p_1 and r_1 (respectively, p_2) (holding the rivalâ€™s prices fixed), and also, the solution to the firstâ€‘order conditions is strictly interior (i.e., positive), given the assumption of non-negative prices (Assumption 2), the interior solution constitutes the unique global maximum.

Equation (B.17) shows that the profit of firm 2 is strictly increasing in application price r_2. Thus, firm 2 sets r_2 to extract as much consumer surplus as possible, while also ensuring that all multi-product users consume application A_2, i.e., users obtain a non-negative payoff from consuming application A_2. Using the application demand function (defined by Equation (16)), we have

$$r_2^{NC} = W(z). \tag{B.21}$$

The equilibrium demand is obtained by substituting Equations (B.18), (B.19), and (B.20) in

Equations (A.9) and (A.10) to get

$$N_1^{NC} = \frac{1}{2} + \frac{\alpha W(z)}{6t} - \frac{\alpha zV}{6t} - \frac{\alpha zq}{6t},\ N_2^{NC} = 1 - N_2^{NC};\ \ a_1^{NC} = a_2^{NC} = 1;$$
$$X_1^{NC} = D_1^{NC} = \alpha\left[\frac{1}{2} + \frac{(3-\alpha)W(z)}{12t} - \frac{(3-\alpha)zV}{12t} - \frac{(3-2\alpha)zq}{6t}\right],\ \ X_2^{NC} = D_1 + D_2 = \alpha,$$

and the equilibrium advertising level is $a_1^{NN} = a_2^{NN} = 1$. Combining this with Equation (B.13), we have

$$s_1^{NC} = \frac{(1-2z)qD_1^{NC} + q(N_1^{NC} - D_1^{NC})}{N_1^{NC}},\ \textit{and}\ s_2 = \frac{(1-z)qD_2^{NC} + q(N_2^{NC} - D_2^{NC})}{N_2^{NC}}. \tag{B.22}$$

Using Equation (1), firms' profit are $\pi_1^{NC} = (p_1^{NC} + q)N_1^{NC} + (r_1^{NC} - 2zq)D_1^{NC}$, and $\pi_2^{NC} = (p_2^{NC} + q)N_2^{NC} + r_2^{NC}\alpha - zqD_2^{NC} - F$. This completes the proof. ■

Proof of Proposition 1

We establish the comparison of prices through a series of steps. To proceed, without loss of generality, suppose that under asymmetric compatibility regime, firm 1 chooses to make its application A_1 incompatible with rival firmâ€™s hardware H_2, whereas firm 2 chooses to make its application A_2 compatible with rival firmâ€™s hardware H_1. In *Step 1*, we compare hardware prices under different market regimes to show that $p_2^{NC} \leq p_1^{NC} \leq p_1^{NN} = p_2^{NN} \leq p_1^{CC} = p_2^{CC}$. In *Step 2*, we compare application prices under different market regimes to show that $r_1^{NN} = r_2^{NN} < r_1^{NC} < r_2^{CN} = r_1^{CC} = r_2^{CC}$. In *Step 3*, we compare the advertising prices under different market regimes to show that $s_1^{NC} < s_1^{CC} = s_2^{CC} < s_1^{NN} = s_2^{NN} < s_2^{NC}$.

Step 1: Comparison of hardware prices.

Using Equations (B.4) and (B.10), we find that the symmetric hardware price is lower for both firms in the incompatibility regime than in the full compatibility regime, i.e.,

$$t - q\ = p_1^{NN} = p_2^{NN}\ \leq\ p_1^{CC} = p_2^{CC}\ =\ t - q + 2\alpha zq, \tag{B.23}$$

with strict inequality for all $\alpha \in (0, 1]$. Using Equations (B.18) and (B.19), we compare the hardware prices of the firms in the asymmetric compatibility regimes, which gives

$$p_1^{NC} == t - q - \frac{\alpha W(z)}{6} + \frac{\alpha z V}{6} + \frac{4\alpha z q}{6} \geq t - q - \frac{\alpha W(z)}{3} + \frac{\alpha z V}{3} + \frac{4\alpha z q}{3} = p_2^{NC}, \quad \text{(B.24)}$$

given the assumption of full market coverage (Assumption 1). Now, combining Equations (B.23) and (B.24), we have

$$p_2^{NC} \leq p_1^{NC} \leq p_1^{NN} = p_2^{NN} \leq p_1^{CC} = p_2^{CC}, \quad \text{(B.25)}$$

with strict inequality for all $\alpha \in (0, 1]$, given the assumption of non-negative prices (Assumption 2).[B.1]

Step 2: Comparison of application prices.

Using Equations (B.5) and (B.11), we find that the symmetric application price is lower for both firms in the incompatibility regime than in the full compatibility regimes, i.e.,

$$r_1^{NN} = r_2^{NN} = zq \leq W(z) = r_1^{CC} = r_2^{CC}. \quad \text{(B.26)}$$

Using Equations (B.20) and (B.21), we compare application prices of the firms in the asymmetric compatibility regimes, which gives

$$r_1^{NC} = \frac{W(z)}{2} - \frac{zV}{2} + zq \leq W(z) = r_2^{NC}, \quad \text{(B.27)}$$

given the assumption of full market coverage (Assumption 1). Combining Equations (B.26) and (B.27), we have

$$r_1^{NN} = r_2^{NN} < r_1^{NC} < r_2^{NC} = r_1^{CC} = r_2^{CC},$$

given the assumption of full market coverage (Assumption 1).[B.2]

[B.1] Equivalently, consider the asymmetric compatibility regime CN, where firm 1 chooses to make its application A_1 compatible with rival firmâ€™s hardware H_2, whereas firm 2 chooses to make its application A_2 incompatible with rival firmâ€™s hardware H_1. Using the same argument, we can show that $p_1^{CN} \leq p_2^{CN} \leq p_1^{NN} = p_2^{NN} \leq p_1^{CC} = p_2^{CC}$.

[B.2] Equivalently, consider the asymmetric compatibility regime CN, where firm 1 chooses to make its application A_1 compatible with rival firmâ€™s hardware H_2, whereas firm 2 chooses to make its application A_2 incompatible

Step 3: Comparison of advertising prices.

Using Equations (B.6) and (B.12), we find that the symmetric advertising price is higher for both firms in the incompatibility regime than in the full compatibility regime, i.e.,

$$s_1^{NN} = s_2^{NN} \; = \; q - \alpha z q \; \geq \; q - 2\alpha z q \; = \; s_1^{CC} = s_2^{CC}, \tag{B.28}$$

with strict inequality for all $\alpha \in (0, 1]$. Next, we compare prices under asymmetric compatibility regime NC with incompatibility regime NN and full compatibility regime CC through a series of claims.

Claim 1. *Under asymmetric compatibility regime, $\bar{x} > \hat{x}$, i.e., the location of the multi-product user indifferent between consuming incompatible firm 1's hardware H_1 and compatible firm 2's hardware H_2 (determined by $\bar{x}$), is to the right of the location of the single-product user indifferent between consuming incompatible firm 1's hardware H_1 and compatible firm 2's hardware H_2 (determined by $\hat{x}$).*

Proof: Without loss of generality, in the asymmetric incompatibility regime NC, using Equations (A.1) and (A.8), we have

$$\begin{aligned}
\bar{x} - \hat{x} &= \left[\frac{1}{2} + \frac{p_2^{NC} - p_1^{NC}}{2t} - \frac{r_1^{NC}}{2t} + \frac{W(z)}{2t} - \frac{zV}{2t}\right] - \left[\frac{1}{2} + \frac{p_2^{NC} - p_1^{NC}}{2t}\right] \\
&= -\frac{r_1^{NC}}{2t} + \frac{W(z)}{2t} - \frac{zV}{2t} \\
&= -\frac{\frac{W(z)}{2} - \frac{zV}{2} + zq}{2t} + \frac{W(z)}{2t} - \frac{zV}{2t} \qquad \text{(by using } r_1^{NC} \text{ defined by Equation (B.20))} \\
&= \frac{W(z)}{4t} - \frac{zV}{4t} + \frac{zq}{2t} \; > \; 0,
\end{aligned}$$

where the last inequality holds given the assumption of full market coverage (Assumption 1). ■

Claim 2. $s_1^{CC} = s_2^{CC} > s_1^{NC}$.

Proof: Using Equations (B.12) and (B.22), we have $s_1^{NC} = \frac{(1-2z)qD_1^{NC} + q(N_1^{NC} - D_1^{NC})}{N_1^{NC}}$, and $s_1^{CC} = s_2^{CC} =$

with rival firmâ€™s hardware H_1. Using the same argument, we can show that $r_1^{NN} = r_2^{NN} < r_2^{CN} < r_1^{CN} = r_1^{CC} = r_2^{CC}$.

$q - 2\alpha z q$. Using these expressions, we have

$$
\begin{aligned}
q - 2\alpha z q - \left[\frac{(1-2z)qD_1^{NC} + q(N_1^{NC} - D_1^{NC})}{N_1^{NC}}\right] &= \frac{1}{N_1^{NC}}[(q - 2\alpha z q)N_1^{NC} - (1-2z)qD_1^{NC} - q(N_1^{NC} - D_1^{NC})] \\
&= \frac{1}{N_1^{NC}}[(-2\alpha z q N_1^{NC} + 2zqD_1^{NC}] \\
&= \frac{2zq}{N_1^{NC}}[(-\alpha N_1^{NC} + D_1^{NC}] \\
&= \frac{2z}{N_1^{NC}}[-\alpha[(1-\alpha)\bar{x} + \alpha\hat{x}] + \alpha\bar{x}] \quad \text{(By using (A.9) and (A.10))} \\
&= \frac{2z\alpha}{N_1^{NC}}[\alpha(\bar{x} - \hat{x})] > 0,
\end{aligned}
$$

where the last inequality follows from Claim 1. ■

Claim 3. $s_2^{NC} > s_1^{NN} = s_2^{NN}$.

Proof: Using Equations (B.12) and (B.22), we have $s_1^{NC} = \frac{(1-z)qD_2^{NC} + q(N_2^{NC} - D_2^{NC})}{N_2^{NC}}$,and $s_1^{NN} = s_2^{NN} = q - \alpha z q$. Using these expressions we have

$$
\begin{aligned}
q - \alpha z q - \left[\frac{(1-z)qD_2^{NC} + q(N_2^{NC} - D_2^{NC})}{N_2^{NC}}\right] &= \frac{1}{N_2^{NC}}[(q - \alpha z q)N_2^{NC} - [(1-z)qD_2^{NC} + q(N_2^{NC} - D_2^{NC})]] \\
&= \frac{1}{N_2^{NC}}[-\alpha z q N_2^{NC} + zqD_2^{NC}] \\
&= \frac{zq}{N_2^{NC}}[-\alpha N_2^{NC} + D_2^{NC}] \quad \text{(By using (A.9) and (A.10))} \\
&= \frac{zq}{N_2^{NC}}[-\alpha[(1-\alpha)(1-\hat{x}) + \alpha(1-\bar{x})] + \alpha(1-\bar{x})] \\
&= \frac{\alpha z q}{N_2^{NC}}[(1-\alpha)(1-\bar{x}) - (1-\alpha)(1-\hat{x})] \\
&= \frac{\alpha z q}{N_2^{NC}}[(1-\alpha)(-\bar{x} + \hat{x})] < 0,
\end{aligned}
$$

where the last inequality follows from Claim 1. ■

Using Equation (B.28) and Claims 2 and 3, we have

$$s_1^{NC} < s_1^{CC} = s_2^{CC} < s_1^{NN} = s_2^{NN} < s_2^{NC},$$

given the assumption of full market coverage (Assumption 1).[B.3] This completes the proof. ■

Proof of Proposition 2

We proceed through a series of steps. In *Step 1*, we characterize the conditions for the adoption of compatibility by the firms. In *Step 2*, we compare the thresholds obtained from *Step 1*. In *Step 3*, we characterize firms' optimal strategies and show that there exist multiple equilibria.

Step 1: Conditions for choosing compatibility by the firms.

Suppose firm $i \in \{1, 2\}$ believes that firm $j \neq i$ chooses incompatibility. Then, using Lemmas 1 and 3, we find that firm i is indifferent between choosing incompatibility and compatibility when compatibility cost is $F_1(\alpha)$, such that (i) $\pi_i^{NN}(F) = \pi_i^{CN}(F)$, at $F = F_1(\alpha)$, (ii) $\pi_i^{NN}(F) < \pi_i^{CN}(F)$, for all $F < F_1(\alpha)$, and (iii) $\pi_i^{NN}(F) > \pi_i^{CN}(F)$, for all $F > F_1(\alpha)$, where $F_1(\alpha)$ is obtained by solving

$$\begin{aligned}\frac{t}{2} = & \left[t - \frac{\alpha W(z)}{3} + \frac{\alpha z V}{3} + \frac{4\alpha z q}{3}\right] \cdot \left[\frac{1}{2} - \frac{\alpha W(z)}{6t} + \frac{\alpha z V}{6t} + \frac{\alpha z q}{6t}\right] + \alpha W(z) \\ & - \alpha z q \left[\frac{1}{2} - \frac{(3-\alpha)W(z)}{12t} + \frac{(3-\alpha)zV}{12t} + \frac{(3-2\alpha)zq}{6t}\right] - F_1(\alpha),\end{aligned}$$

$$\begin{aligned}F_1(\alpha) = & \left[t - \frac{\alpha W(z)}{3} + \frac{\alpha z V}{3} + \frac{4\alpha z q}{3}\right] \cdot \left[\frac{1}{2} - \frac{\alpha W(z)}{6t} + \frac{\alpha z V}{6t} + \frac{\alpha z q}{6t}\right] + \alpha W(z) \\ & - \alpha z q \left[\frac{1}{2} - \frac{(3-\alpha)W(z)}{12t} + \frac{(3-\alpha)zV}{12t} + \frac{(3-2\alpha)zq}{6t}\right] - \frac{t}{2}.\end{aligned} \tag{B.29}$$

In summary, $F_1(\alpha)$ is the threshold on compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application incompatible.

Suppose firm $i \in \{1, 2\}$ believes that firm $j \neq i$ chooses compatibility. Then, using Lemmas 2

[B.3] Equivalently, consider the asymmetric compatibility regime, where firm 1 chooses to make its application A_1 compatible with rival firmâ€™s hardware H_2, whereas firm 2 chooses to make its application A_2 incompatible with rival firmâ€™s hardware H_1. Using the same argument, we can show that $s_2^{CN} < s_1^{CC} = s_2^{CC} < s_1^{NN} = s_2^{NN} < s_1^{CN}$.

and 3, we find that firm i is indifferent between choosing incompatibility and compatibility when its compatibility cost is $F_2(\alpha)$, such that (i) $\pi_i^{NC}(F) = \pi_i^{CC}(F)$, at $F = F_2(\alpha)$, (ii) $\pi_i^{NC}(F) < \pi_i^{CC}(F)$, for all $F < F_2(\alpha)$, and (iii) $\pi_i^{NC}(F) > \pi_i^{CC}(F)$, for all $F > F_2(\alpha)$, where $F_2(\alpha)$ is obtained by solving

$$\frac{t}{2} + \alpha W(z) - F_2(\alpha) = \left[t - \frac{\alpha W(z)}{6} + \frac{\alpha z V}{6} + \frac{4\alpha z q}{6}\right] \cdot \left[\frac{1}{2} + \frac{\alpha W(z)}{6t} - \frac{\alpha z V}{6t} - \frac{\alpha z q}{6t}\right] + \alpha \left[\frac{W(z)}{2} - \frac{zV}{2} - zq\right] \cdot \left[\frac{1}{2} + \frac{(3-\alpha)W(z)}{12t} - \frac{(3-\alpha)zV}{12t} - \frac{(3-2\alpha)zq}{6t}\right],$$

$$F_2(\alpha) = \frac{t}{2} + \alpha W(z) - \left[t - \frac{\alpha W(z)}{6} + \frac{\alpha z V}{6} + \frac{4\alpha z q}{6}\right] \cdot \left[\frac{1}{2} + \frac{\alpha W(z)}{6t} - \frac{\alpha z V}{6t} - \frac{\alpha z q}{6t}\right] - \alpha \left[\frac{W(z)}{2} - \frac{zV}{2} - zq\right] \cdot \left[\frac{1}{2} + \frac{(3-\alpha)W(z)}{12t} - \frac{(3-\alpha)zV}{12t} - \frac{(3-2\alpha)zq}{6t}\right]. \tag{B.30}$$

In summary, $F_2(\alpha)$ is the threshold on compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application compatible.

Step 2: Comparison of thresholds $F_1(\alpha)$ and $F_2(\alpha)$.

Claim 4. $\frac{\partial F_1(\alpha)}{\partial \alpha} > 0$, *for all* $\alpha > 0$.

Proof. Using Equation (B.29), and differentiating $F_1(\alpha)$ with respect to α gives

$$\frac{\partial F_1(\alpha)}{\partial \alpha} = -\left[\frac{W(z)}{3} - \frac{zV}{3} - \frac{4zq}{3}\right] \cdot N_i^{CN} - \left[t - \frac{\alpha W(z)}{3} + \frac{\alpha z V}{3} + \frac{4\alpha z q}{3}\right] \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t} - \frac{zq}{6t}\right] + W(z) - zq \cdot \left[\frac{1}{2} - \frac{(3-2\alpha)W(z)}{12t} + \frac{(3-2\alpha)zV}{12t} + \frac{(3-4\alpha)2zq}{12t}\right]. \tag{B.31}$$

Let us define

$$B := W(z) - zq \cdot \left[\frac{1}{2} - \frac{(3-2\alpha)W(z)}{12t} + \frac{(3-2\alpha)zV}{12t} + \frac{(3-4\alpha)2zq}{12t}\right].$$

From Lemma 3, note that $N_i^{CN} \leq \frac{1}{2}$ and $p_i^{CN} \leq t$. Using these inequalities and the expression for $\frac{\partial F_1(\alpha)}{\partial \alpha}$, we obtain

$$\begin{aligned}\frac{\partial F_1(\alpha)}{\partial \alpha} &\geq B - \frac{1}{2} \cdot \left[\frac{W(z)}{3} - \frac{zV}{3} - \frac{4zq}{3}\right] - t \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t} - \frac{zq}{6t}\right] \\ &= B - \frac{1}{6} \cdot \left[2W(z) - 2zV - 5zq\right] \\ &= \frac{W(z)}{3} + \frac{zV}{3} + \frac{zq}{3} + zq \cdot \left[\frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-4\alpha)2zq}{12t}\right] \;>\; 0,\end{aligned}$$

where the final inequality holds for all $\alpha > 0$, given the assumption of full market coverage (Assumption 1). Hence, we have $\frac{\partial F_1(\alpha)}{\partial \alpha} > 0$, for all $\alpha > 0$. ■

Claim 5. $\frac{\partial F_2(\alpha)}{\partial \alpha} > 0$, *for all* $\alpha > 0$.

Proof. Using Equation (B.30), and differentiating $F_2(\alpha)$ with respect to α gives

$$\begin{aligned}\frac{\partial F_2(\alpha)}{\partial \alpha} \;=\; & W(z) - \left[-\frac{W(z)}{6} + \frac{zV}{6} + \frac{2zq}{3}\right] \cdot N_i^{NC} \\ & - \left[t - \frac{\alpha W(z)}{6} + \frac{\alpha zV}{6} + \frac{4\alpha zq}{6}\right] \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t} - \frac{zq}{6t}\right] \\ & - \left[\frac{W(z)}{2} - \frac{zV}{2} - zq\right] \cdot \left[\frac{1}{2} + \frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-4\alpha)2zq}{12t}\right].\end{aligned} \tag{B.32}$$

Let us define

$$B := \left[\frac{W(z)}{2} - \frac{zV}{2} - zq\right]$$

From Lemma 3, note that $N_i^{NC} \geq \frac{1}{2}$ and $p_i^{NC} \leq t$. Using these inequalities and the expression for $\frac{\partial F_2(\alpha)}{\partial \alpha}$, we obtain

$$\begin{aligned}\frac{\partial F_2(\alpha)}{\partial \alpha} &> W(z) + \frac{1}{2} \cdot \left[\frac{W(z)}{6} - \frac{zV}{6} - \frac{2zq}{3}\right] - \left[\frac{W(z)}{6} - \frac{zV}{6} - \frac{zq}{6}\right] \\ &\quad - \frac{1}{2} \cdot B - B\left[\frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-4\alpha)2zq}{12t}\right] \;>\; 0 \\ &= \frac{2W(z)}{3} + \frac{zV}{3} + \frac{zq}{3} - B\left[\frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-4\alpha)2zq}{12t}\right] \\ &> \frac{2W(z)}{3} + \frac{zV}{3} + \frac{zq}{3} - B\left[\frac{W(z)}{4t} - \frac{zV}{4t} - \frac{2zq}{4t}\right] \\ &= \frac{2W(z)}{3} + \frac{zV}{3} + \frac{zq}{3} - \left[\frac{W(z)}{2} - \frac{zV}{2} - zq\right]\left[\frac{W(z)}{4t} - \frac{zV}{4t} - \frac{2zq}{4t}\right] > 0,\end{aligned}$$

where the second-last inequality follows because $\left[\frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-4\alpha)2zq}{12t}\right]$ is strictly decreasing in α, and last inequality follows because $t > \frac{3[W(z)-z(V-\delta)-2zq]^2}{8[2W(z)+z(V-\delta)=zq]}$, given the assumption of non-negative prices (Assumption 2). ■

Claim 6. *$\frac{\partial F_1(\alpha)}{\partial \alpha} > \frac{\partial F_2(\alpha)}{\partial \alpha}$, for all $\alpha > 0$.*

Proof. Note that since firms are symmetric, $N_i^{CN} = 1 - N_i^{NC}$. Also, $N_i^{NC} > \frac{1}{2}$, and $\bar{x} > \frac{1}{2}$ (where $\bar{x}$ is defined by Equation (A.8)). Combining this with Equations (B.31) and (B.32), we have

$$
\begin{aligned}
\frac{\partial F_1(\alpha)}{\partial \alpha} - \frac{\partial F_2(\alpha)}{\partial \alpha} = & \left[\frac{W(z)}{6} - \frac{zV}{6} - \frac{2zq}{3}\right] \cdot N_i^{NC} - \left[\frac{W(z)}{3} - \frac{zV}{3} - \frac{zq}{3}\right] \\
& + \left[\frac{\alpha W(z)}{6} - \frac{\alpha zV}{6} - \frac{4\alpha zq}{6}\right] \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t} - \frac{zq}{6t}\right] \\
& + \left[\frac{W(z)}{2} - \frac{zV}{2}\right] \cdot \left[\frac{1}{2} + \frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-4\alpha)2zq}{12t}\right] \\
> & \left[\frac{W(z)}{6} - \frac{zV}{6} - \frac{2zq}{3}\right] \cdot \frac{1}{2} + \left[\frac{W(z)}{2} - \frac{zV}{2}\right] \cdot \frac{1}{2} - \left[\frac{W(z)}{3} - \frac{zV}{3} - \frac{zq}{3}\right] \\
+ & \left[\frac{\alpha W(z)}{6} - \frac{\alpha zV}{6} - \frac{4\alpha zq}{6}\right] \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t} - \frac{zq}{6t}\right] \\
= & \left[\frac{\alpha W(z)}{6} - \frac{\alpha zV}{6} - \frac{4\alpha zq}{6}\right] \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t} - \frac{zq}{6t}\right] \geq 0,
\end{aligned}
$$

where the final inequality holds for all values of $\alpha \geq 0$, given the assumption of full market coverage (Assumption 1). ■

Finally, note that, at $\alpha = 0$, the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ take the same values, $F_1(0) = F_2(0) = 0$. Combining this with Claims 4, 5, and 6, we have $F_1(\alpha) \geq F_2(\alpha)$ for all $\alpha \in [0, 1]$.

Step 3: Optimal strategies and equilibrium outcomes.

Now, we use the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ on compatibility costs, to characterize firms' optimal strategies. For sufficiently small compatibility costs, i.e., $F < F_2(\alpha)$, it is optimal for a firm to choose compatibility, irrespective of its belief about the rival firm's choice. Thus, the dominant strategy for each firm is to choose compatibility. This implies that, at *Stage 1*, the Nash equilibrium - the firms' best responses to each other's individual optimal strategies - is given by both firms choosing compatibility, i.e., regime CC. For sufficiently large compatibility costs, i.e., $F \geq F_1(\alpha)$, it is optimal for a firm to choose incompatibility, irrespective of its belief about the rival firm's choice. Thus, the dominant strategy for each firm is to choose incompatibility. This implies that, at *Stage 1*, the Nash equilibrium is given by both firms choosing incompatibility, i.e., regime NN. For the intermediate range of compatibility costs, i.e., $F_2(\alpha) \leq F < F_1(\alpha)$, it is optimal for a firm to choose compatibility if it believes that its rival chooses incompatibility, and it is optimal for a firm to choose incompatibility if it believes that its rival chooses compatibility. This implies that, at *Stage 1*, we have two Nash equilibria: (i) firm 1 choosing incompatibility and firm 2 choosing compatibility, i.e., regime NC, or (ii) firm 1 choosing compatibility and firm 2 choosing incompatibility, i.e., regime CN. This completes the proof. ■

Proof of Proposition 3

Proposition 2 characterized the region in which each regime is preferred by firms, using thresholds $F_1(\alpha)$ and $F_2(\alpha)$ on compatibility costs that firms face. In this proposition, we examine the change in the parameter space where each regime is the equilibrium outcome, with respect to model parameters. We proceed through a series of steps. In *Step 1*, we characterize the sensitivity of the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ with respect to per-unit transportation cost t. In *Step 2*, we characterize the sensitivity of the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ with respect to the advertiser revenue per user per-unit of time q.

Step 1: Impact of per-unit transportation cost t on the thresholds $F_1(\alpha)$ and $F_2(\alpha)$.

First, consider threshold $F_1(\alpha)$. Using Equation (B.29), we have

$$\begin{aligned}\frac{\partial F_1(\alpha)}{\partial t} &= -\frac{\left[\alpha W(z) - \alpha z V - 4\alpha z q\right] \cdot \left[\alpha W(z) - \alpha z V - \alpha z q\right]}{18t^2} \\ &\quad - \alpha z q \cdot \left[\frac{(3-\alpha)W(z) - (3-\alpha)zV + 2(3-2\alpha)zq}{12t^2}\right] \leq 0,\end{aligned} \tag{B.33}$$

where the final inequality holds for all $\alpha \geq 0$, given the assumption of full market coverage (Assumption 1).

Next, consider threshold $F_2(\alpha)$. Using Equation (B.30), we have

$$\begin{aligned}\frac{\partial F_2(\alpha)}{\partial t} &= -\frac{\left[\alpha W(z) - \alpha zV - 4\alpha zq\right] \cdot \left[\alpha W(z) - \alpha zV - \alpha zq\right]}{36t^2} \\ &\quad + \alpha\left[\frac{W(z)}{2} - \frac{zV}{2} - zq\right] \cdot \left[\frac{(3-\alpha)W(z) - (3-\alpha)zV + 2(3-2\alpha)zq}{12t^2}\right] \\ &\geq -\alpha\frac{\left[W(z) - zV - zq\right]^2}{36t^2} + \alpha\left[\frac{3W(z)}{2} - \frac{3zV}{2} - 3zq\right] \cdot \left[\frac{3-\alpha)W(z) - (3-\alpha)zV + 2(3-2\alpha)zq}{36t^2}\right].\end{aligned} \tag{B.34}$$

Let us define

$$A := W(z) - zV - zq, \quad B := \frac{3W(z)}{2} - \frac{3zV}{2} - 3zq, \text{ and } C := (3-\alpha)W(z) - (3-\alpha)zV + 2(3-2\alpha)zq.$$

Now, using the values of A, B, and C, we have

$$\begin{aligned}B - A &= \frac{3W(z)}{2} - \frac{3zV}{2} - 3zq - W(z) + zV + zq \geq 0, \;\; \textit{and} \\ C - A &= (3-\alpha)W(z) - (3-\alpha)zV + 2(3-2\alpha)zq - W(z) + zV + zq \geq 0,\end{aligned} \tag{B.35}$$

where the inequalities holds for all $\alpha \in [0,1]$, given the assumption of full market coverage (Assumption 1). Therefore, using the inequalities established by Equation (B.35), we have $B \geq A$, and $C \geq A$, which in turn implies $B.C \geq A^2$. Using this inequality, we can argue that $\frac{\partial F_2(\alpha)}{\partial t} > 0$. This shows that the threshold $F_1(\alpha)$ (respectively, $F_2(\alpha)$) on compatibility cost between incompatibility NN (full compatibility regime CC) and asymmetric compatibility regimes, decreases (respectively, increases) with an increase in per-unit transportation cost t. Moreover, using Equations (B.33) and (B.34), since $\frac{\partial F_2(\alpha)}{\partial t} > 0$, and $\frac{\partial F_1(\alpha)}{\partial t} < 0$, we have $\frac{\partial F_2(\alpha)}{\partial t} - \frac{\partial F_1(\alpha)}{\partial t} > 0$.

Therefore, as per-unit transportation cost increases, the parameter space over which firms choose incompatibility and full compatibility in equilibrium increases. Further, the region with $F_2(\alpha) \leq F \leq F_1(\alpha)$, where asymmetric compatibility regimes arise as an equilibrium becomes smaller. As an illustration (refer Figure B.1), we use Example 1 with parameter values: $V = 2$, $w = 4$, $q = 0.7$, $z = 0.3$, $b = 0.5$, and $t \in \{1,2\}$, and show the change in thresholds $F_1(\alpha)$ and $F_2(\alpha)$ as t increases from 1 to 2.

Step 2: Impact of advertiser revenue per user per-unit of time q on the thresholds $F_1(\alpha)$ and $F_2(\alpha)$.

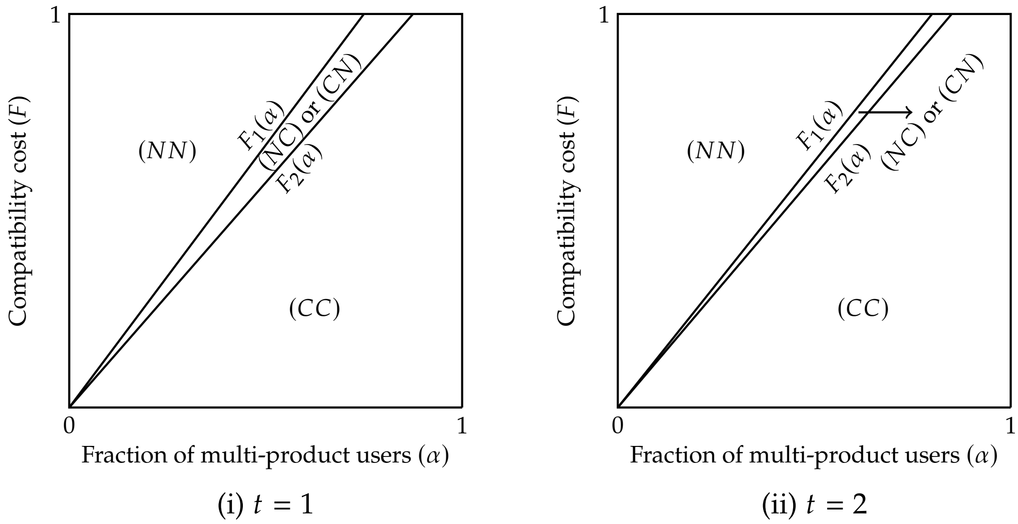


Figure B.1: Comparative statics of optimal compatibility regimes with respect to per-unit transportation cost t.

The figure is based on Example 1 with parameter values $V = 2, w = 4, q = 0.7, z = 0.3$ and $b = 0.5$. The threshold $F_1(\alpha)$ (respectively, $F_2(\alpha)$) represents the loci of points along which a firm is indifferent between choosing compatibility and incompatibility when it believes that rival has chosen incompatibility (respectively, compatibility).

First, consider threshold $F_1(\alpha)$. Using Equation (B.29), differentiating w.r.t. q, and after algebraic calculations, we obtain

$$\begin{aligned}\frac{\partial F_1(\alpha)}{\partial q} &= \frac{\alpha z}{36t}[12t + 4(-9 + 10\alpha)qz + (-9 + 13\alpha)(zV - W(z))] \\ &> \frac{\alpha z}{36t}[12t + 4qz + 4\alpha(zV - W(z))] > 0,\end{aligned} \tag{B.36}$$

where the first inequality holds because $12t + 4(-9 + 10\alpha)qz + (-9 + 13\alpha)(zV - W(z))$ is strictly decreasing in α for $\alpha \in [0, 1]$, and the second inequality holds because $12t+4qz+4\alpha(zV-W(z)) > 0$, given the assumption of non-negative prices (Assumption 2). Now, consider threshold $F_2(\alpha)$. Using Equation (B.30), we have

$$\begin{aligned}\frac{\partial F_2(\alpha)}{\partial q} &= \frac{\alpha z}{18t}[6t + 2(-9 + 8\alpha)qz + (-9 + 7\alpha)(zV - W(z))] \\ &> \frac{\alpha z}{18t}[6t - 2qz - 2(zV - W(z))] > 0,\end{aligned} \tag{B.37}$$

where the first inequality holds because $6t + 2(-9 + 8\alpha)qz + (-9 + 7\alpha)(zV - W(z))$ is strictly decreasing in α for $\alpha \geq 0$, and the second inequality holds because $W(z) - zV - 2qz > 0$ for all values of $\alpha \geq 0$, given the assumption of full market coverage (Assumption 1). This shows that both thresholds on compatibility costs decrease with an increase in q. Moreover, comparing Equations (B.36) and (B.37), we have

$$\begin{aligned}\frac{\partial F_1(\alpha)}{\partial q} - \frac{\partial F_2(\alpha)}{\partial q} &= \frac{\alpha z}{36t}[\alpha(8zq - zV + W(z)) + 9(zV - W(z))] \\ &= \frac{\alpha z}{36t}[\alpha(8zq + 8zV - 8W(z)] < 0,\end{aligned} \tag{B.38}$$

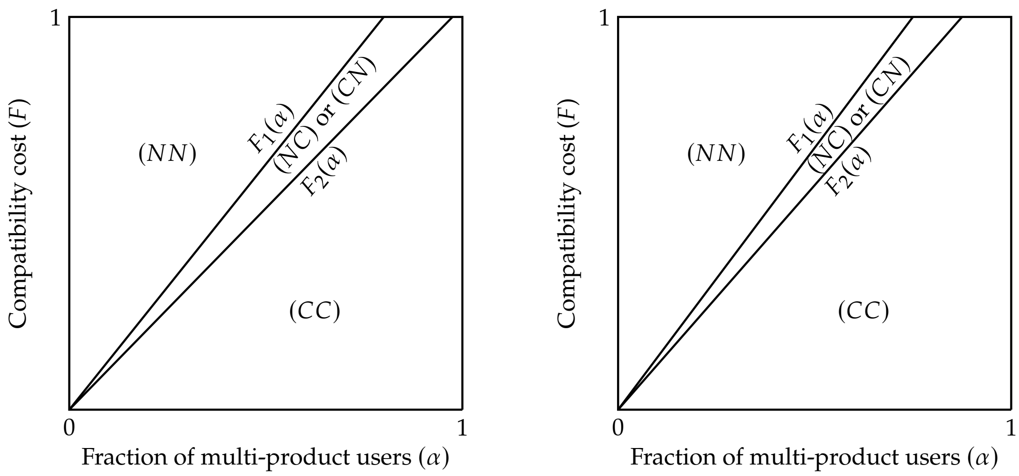
where the final inequality holds because $W(z) - zV - qz > 0$ for all values of $\alpha \geq 0$, given the assumption of full market coverage (Assumption 1).Therefore, as both thresholds increase, the parameter space over which firms choose incompatibility in equilibrium reduces. Further, as threshold $F_2(\alpha)$ increase at a higher rate than thresholds $F_1(\alpha)$, the region with $F_2(\alpha) \leq F < F_1(\alpha)$, where asymmetric compatibility regimes arise in equilibrium becomes smaller. As an illustration (refer Figure B.2), we use Example 1 with numerical values: $V = 2$, $W = 7$, $z = 0.3$, $t = 1$, $b = 0.5$ and $q \in \{0.1, 0.7\}$, and show the change in thresholds $F_1(\alpha)$ and $F_2(\alpha)$ as q increases from 0.1 to 0.7.

This completes the proof. ■

Proof of Proposition 6

We proceed through a series of steps. In *Step 1*, we define social welfare under different market regimes. In *Step 2*, we compare social welfare under different market regimes and derive conditions under which incompatibility, full compatibility, or asymmetric compatibility regime is socially optimal. In *Step 3*, we compare thresholds obtained from *Step 2*. In *Step 4*, we characterize the socially optimal compatibility regimes.

Step 1: Social welfare under different market regimes.



(i) $q = 0.1$ (ii) $q = 0.7$

Figure B.2: Comparative statics of optimal compatibility regimes with respect to fraction of time spent on hardware z.

The figure is based on parameter values $V = 2$, $w = 7$, $z = 0.3$,, $b = 0.5$, and $t = 1$. The threshold $F_1(\alpha)$ ($F_2(\alpha)$) represents the loci of points along which a firm is indifferent between choosing compatibility and incompatibility when it believes that rival has chosen incompatibility (compatibility).

If both firms choose incompatibility, then social welfare is given by

$$\begin{aligned} SW^{NN} &= (1-\alpha)\left[\int_0^{\hat{x}}(V-tx)dx + \int_{\hat{x}}^1 (V-t(1-x))dx\right] \\ &\quad + \alpha\left[\int_0^{\bar{x}}[V(1-z)+W(z)-tx]dx + \int_{\bar{x}}^1 [V(1-z)+W(z)-t(1-x)]dx\right] \\ &\quad + (1-\alpha)q + \alpha q(1-z). \end{aligned}$$

Using Equations (A.1) and (A.2) on the user demand under incompatibility, i.e., regime NN, we obtain

$$SW^{NN} = (1-\alpha)V + \alpha[V(1-z)+W(z)] - \frac{t}{4} + (1-\alpha)q + \alpha q(1-z). \tag{B.39}$$

If both firms choose compatibility, then social welfare is given by

$$\begin{aligned} SW^{CC} &= (1-\alpha)\left[\int_0^{\hat{x}}(V-tx)dx + \int_{\hat{x}}^1 (V-t(1-x))dx\right] \\ &+ \alpha\left[\int_0^{\bar{x}}[V(1-2z)+2W(z)-tx]dx + \int_{\bar{x}}^1 [V(1-2z)+2W(z)-t(1-x)]dx\right] \\ &+ (1-\alpha)q + \alpha q(1-2z) - 2F. \end{aligned}$$

Using Equations (A.1) and (A.5) on the user demand under full compatibility, i.e., regime CC, we obtain

$$SW^{CC} = (1-\alpha)V + \alpha[V(1-2z)+2W(z)] - \frac{t}{4} + (1-\alpha)q + \alpha q(1-2z) - 2F. \quad \text{(B.40)}$$

Without loss of generality, suppose firm 1 chooses incompatibility and firm 2 chooses compatibility, i.e., regime NC. Under an asymmetric compatibility regime, social welfare is given by

$$\begin{aligned} SW^{NC} &= (1-\alpha)\left[\int_0^{\hat{x}}(V-tx)dx + \int_{\hat{x}}^1 (V-t(1-x))dx\right] \\ &+ \alpha\left[\int_0^{\bar{x}}[V(1-2z)+2W(z)-tx]dx + \int_{\bar{x}}^1 [V(1-z)+W(z)-t(1-x)]dx\right] \\ &+ (1-\alpha)q + \alpha q(1-2z)\bar{x} + \alpha q(1-z)(1-\bar{x}) - F. \end{aligned}$$

After algebraic calculations, we obtain

$$\begin{aligned} S^{NC} &= (1-\alpha)V + \alpha[V(1-z-\bar{x}z)+W(z)(1+\bar{x})] - \frac{t}{4} - (1-\alpha)t\left(\frac{1}{2}-\hat{x}\right)^2 - \alpha t\left(\frac{1}{2}-\bar{x}\right)^2 \\ &+ (1-\alpha)q + \alpha q(1-2z)\bar{x} + \alpha q(1-z)(1-\bar{x}) - F. \end{aligned} \quad \text{(B.41)}$$

Step 2: Comparison of social welfare under different market regimes.

Using Equations (B.39) and (B.40), we find that the social planner is indifferent between choosing incompatibility and full compatibility regimes when compatibility cost is $F_{S1}(\alpha)$, such that (i) $SW^{CC}(F) = SW^{NN}(F)$ at $F = F_{S1}(\alpha)$, (ii) $SW^{CC}(F) > SW^{NN}(F)$, for all $F < F_{S1}(\alpha)$, and

(iii) $SW^{CC}(F) < SW^{NN}(F)$, for all $F > F_{S1}(\alpha)$, where

$$F_{S1}(\alpha) = \frac{\alpha}{2}\left[W(z) - Vz - zq\right]. \tag{B.42}$$

In summary, $F_{S1}(\alpha)$ is the threshold on compatibility cost below which a social planner prefers full compatibility over incompatibility.

Using Equations (B.40) and (B.41), we find that the social planner is indifferent between choosing full compatibility and asymmetric compatibility regimes when compatibility cost is $F_{S2}(\alpha)$, such that (i) $SW^{CC}(F) = SW^{NC}(F)$, at $F = F_{S2}(\alpha)$, (ii) $SW^{CC}(F) > SW^{NC}(F)$, for all $F < F_{S2}(\alpha)$, and (iii) $SW^{CC}(F) < SW^{NC}(F)$, for all $F > F_{S2}(\alpha)$, where

$$F_{S2}(\alpha) = \alpha(1 - \bar{x}^{NC})[W(z) - Vz - zq] + (1-\alpha)t\left(\frac{1}{2} - \hat{x}\right)^2 + \alpha t\left(\frac{1}{2} - \bar{x}\right)^2. \tag{B.43}$$

In summary, $F_{S2}(\alpha)$ is the threshold on compatibility below which a social planner prefers full compatibility regime CC over asymmetric compatibility regime NC or CN.

Using Equations (B.39) and (B.41), we find that the social planner is indifferent between choosing incompatibility and asymmetric compatibility when compatibility cost is $F_{S3}(\alpha)$, such that (i) $SW^{NN}(F) = SW^{NC}(F)$, at $F = F_{S3}(\alpha)$, (ii) $SW^{NN}(F) < SW^{NC}(F)$, for all $F < F_{S3}(\alpha)$, and (iii) $SW^{NN}(F) > SW^{NC}(F)$, for all $F > F_{S3}(\alpha)$, where

$$F_{S3}(\alpha) = \alpha\bar{x}^{NC}[W(z) - Vz - zq] - (1-\alpha)t\left(\frac{1}{2} - \hat{x}\right)^2 - \alpha t\left(\frac{1}{2} - \bar{x}\right)^2. \tag{B.44}$$

In summary, $F_{S3}(\alpha)$ is the threshold on compatibility cost above which a social planner prefers incompatibility regime NN over asymmetric compatibility regime NC or CN.

Step 3: Comparison of thresholds $F_{S1}(\alpha)$, $F_{S2}(\alpha)$ and $F_{S3}(\alpha)$.

We establish the comparison of thresholds through a series of claims.

Claim 7. *$F_{S3}(\alpha) \geq F_{S2}(\alpha)$.*

Proof. Let us define $T := (1-\alpha)t\left(\frac{1}{2} - \hat{x}\right)^2 + \alpha t\left(\frac{1}{2} - \bar{x}\right)^2$. Using Equations (B.43) and (B.44), we have

$$\begin{aligned} F_{S3}(\alpha) - F_{S2}(\alpha) &= \alpha(2\bar{x}^{NC} - 1)[W(z) - Vz - zq] - 2T \\ &> \alpha(2\bar{x}^{NC} - 1)[W(z) - Vz - zq] - 2t\left(\frac{1}{2} - \hat{x}\right)^2 \\ &= \alpha(2\bar{x}^{NC} - 1)[W(z) - Vz - zq] - 2t[W(z) - Vz - zq]^2 \frac{\alpha^2}{144t} > 0. \end{aligned} \tag{B.45}$$

where the second last inequality follows because $T < t\left(\frac{1}{2} - \hat{x}\right)^2$, and the last inequality follows because $\bar{x} - \frac{1}{2} > [W(z) - Vz - zq]\frac{\alpha}{144t}$, given the assumption of full market coverage (Assumption 1). ■

Claim 8. $F_{S3}(\alpha) \geq F_{S1}(\alpha)$.

Proof. Let us define $T := (1-\alpha)t\left(\frac{1}{2} - \hat{x}\right)^2 + \alpha t\left(\frac{1}{2} - \bar{x}\right)^2$. Using Equations (B.42) and (B.44), we have

$$F_{S3}(\alpha) - F_{S1}(\alpha) = \alpha\left(\bar{x}^{NC} - \frac{1}{2}\right)[W(z) - Vz - zq] - T > 0, \tag{B.46}$$

where the last inequality follows from Claim 7. ■

Claim 9. $F_{S1}(\alpha) \geq F_{S2}(\alpha)$.

Proof. Let us define $T := (1-\alpha)t\left(\frac{1}{2} - \hat{x}\right)^2 + \alpha t\left(\frac{1}{2} - \bar{x}\right)^2$. Using Equations (B.42) and (B.43), we have

$$F_{S1}(\alpha) - F_{S2}(\alpha) = \alpha\left(\bar{x}^{NC} - \frac{1}{2}\right)[W(z) - Vz - zq] - T > 0, \tag{B.47}$$

where the last inequality follows from Claim 7. ■

Therefore, using Claims 7, 8, and 9, the thresholds can be ordered as follows:

$$F_{S2}(\alpha) \leq F_{S1}(\alpha) \leq F_{S3}(\alpha). \tag{B.48}$$

Step 4: Socially optimal compatibility regimes.

Now, we use inequality defined by Equation (B.48) and thresholds $F_{S1}(\alpha)$, $F_{S2}(\alpha)$, and $F_{S3}(\alpha)$ on compatibility cost, to characterize the socially optimal compatibility regimes. For a sufficiently small compatibility cost, i.e., $F < F_{S2}(\alpha)$, both firms choose compatibility, i.e., regime CC, is socially optimal. For a sufficiently large compatibility costs, i.e., $F \geq F_{S3}(\alpha)$, both firms choose incompatibility, i.e., regime NN, is socially optimal. For intermediate range of compatibility costs, i.e., $F_{S2}(\alpha) \leq F < F_{S3}(\alpha)$, firm 1 chooses incompatibility while firm 2 chooses compatibility, i.e., regime NC, or firm 1 chooses compatibility while firm 2 chooses incompatibility, i.e., regime CN, is socially optimal. This completes the proof. ■

C Advertisements in Both Hardware and Application

In this extension, we allow for advertisers to place advertisements in both the hardware and software. Let a_{Hi} and a_{Ai} denote the quantity of advertisements in hardware and application, respectively, of firm i.

C.1 Stages 5, 4 and 3: Demand from single-product and multi-product users, and advertising demand functions

If a single-product user located at $x \in [0, 1]$ purchases hardware H_i at price p_i and views a_i advertisements, then her net utility is as defined by Equation (2), yielding location of indifferent user $\hat{x}$ as defined by Equation (A.1) under the baseline model.

Next, depending on the compatibility regime, we can have four different scenarios for multi-product users.

Incompatibility (NN)

If application A_j is not compatible with rival hardware H_i, where $j \neq i$, then we are in an incompatible regime NN. At *Stage 5*, we obtain the demand X_i for application A_i as defined by Equation (10). At *Stage 4*, a multi-product user's net utility is as defined by Equation (3), yielding location of indifferent multi-product user as defined by Equation (A.2), and demand functions remain the same as under the baseline model (defined by Equation (A.3)). Therefore, the total demand functions for hardware H_1 and H_2 from both single-product and multi-product users remains the same as under baseline model (defined by Equation (A.4)).

At *Stage 3*, given advertising prices $s_{Hi}, s_{Ai}, i \in \{1, 2\}$, an advertiser places an advertisement in hardware $H_i, i \in \{1, 2\}$ if $(1 - z)qD_i + q(N_i - D_i) - s_{Hi}N_i \geq 0$, and an advertiser places an advertisement in application $A_i, i \in \{1,2\}$ if $zqX_i - s_{Ai}X_i \geq 0$. This gives the level of advertisements a_{Hi} in hardware $H_i, i \in \{1,2\}$ and a_{Ai} in application $A_i, i \in \{1,2\}$ as

$$a_{Hi} = \begin{cases} 1, & \text{if } (1-z)qD_i + q(N_i - D_i) \geq s_{Hi}N_i, \text{ and} \\ 0, & \text{otherwise.} \end{cases} \quad \text{and } a_{Ai} = \begin{cases} 1, & \text{if } zq \geq s_{Ai}, \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{C.1}$$

Full compatibility (CC)

If application A_j is compatible with rival hardware H_i, where $j \neq i$, then we are in full compatibility regime CC. At *Stage 5*, we obtain the demand X_i for application A_i as defined by Equation (13). At *Stage 4*, a multi-product user's net utility is as defined by Equation (4), yielding location of indifferent multi-product user as defined by Equation (A.5), and demand functions remain the same as under the baseline model (defined by Equation (A.6)). Therefore, the total demand functions for hardware H_1 and H_2 from both single-product and multi-product users remain the same as under baseline model (defined by Equation (A.7)).

At *Stage 3*, given advertising prices $s_{Hi}, s_{Ai}, i \in \{1,2\}$, an advertiser places an advertisement in hardware $H_i, i \in \{1,2\}$ if $(1-2z)qD_i + q(N_i - D_i) - s_{Hi}N_i \geq 0$, and an advertiser places an advertisement in application $A_i, i \in \{1,2\}$ if $2zqX_i - s_{Ai}X_i \geq 0$. This gives the level of advertisements a_{Hi} in hardware $H_i, i \in \{1,2\}$ and a_{Ai} in application $A_i, i \in \{1,2\}$ as

$$a_{Hi} = \begin{cases} 1, & \text{if } (1-2z)qD_i + q(N_i - D_i) \geq s_{Hi}N_i, \text{ and} \\ 0, & \text{otherwise,} \end{cases} \quad and \;\; a_{Ai} = \begin{cases} 1, & \text{if } 2zq \geq s_{Ai}, \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{C.2}$$

Asymmetric compatibility (NC)

If application A_1 is incompatible with hardware H_2, whereas application A_2 is compatible with hardware H_1, then we are in an asymmetric compatibility regime NC. At *Stage 5*, we obtain the demand X_i for application A_i as defined by Equation (16). At *Stage 4*, a multi-product user's net utility is as defined by Equation (5), yielding location of indifferent multi-product user as defined by Equation (A.8), and demand functions remain the same as under the baseline model (defined by Equation (A.9)). Therefore, the total demand functions for hardware H_1 and H_2 from both single-product and multi-product users remain the same as under baseline model (defined by Equation (A.10)).

At *Stage 3*, given advertising prices $s_{Hi}, s_{Ai}, i \in \{1,2\}$, an advertiser places an advertisement in hardware H_1 if $(1-2z)qD_1 + q(N_1 - D_1) - s_{H1}N_1 \geq 0$, an advertiser places an advertisement in hardware H_2 if $(1-z)qD_2 + q(N_2 - D_2) - s_{H2}N_2 \geq 0$, an advertiser places an advertisement in application A_1 if $zqX_1 - s_{A1}X_1 \geq 0$, and an advertiser places an advertisement in application A_2 if $2zqX_2 - s_{A2}X_2 \geq 0$. This gives the level of advertisements a_{Hi} in hardware $H_i, i \in \{1,2\}$ and a_{Ai} in hardware $A_i, i \in \{1,2\}$ as

$$a_{H1} = \begin{cases} 1, & \text{if } (1-2z)qD_1 + q(N_1 - D_1) \geq s_{H1}N_1, \text{ and} \\ 0, & \text{otherwise,} \end{cases} \quad a_{H2} = \begin{cases} 1, & \text{if } (1-z)qD_2 + q(N_2 - D_2) \geq s_{H2}N_2, \text{ and} \\ 0, & \text{otherwise,} \end{cases}$$
$$a_{A1} = \begin{cases} 1, & \text{if } zq \geq s_{A1}, \text{ and} \\ 0, & \text{otherwise,} \end{cases} \quad \text{and } a_{A2} = \begin{cases} 1, & \text{if } 2zq \geq s_{A2}, \text{ and} \\ 0, & \text{otherwise,} \end{cases} \tag{C.3}$$

C.2 Stage 2: Equilibrium prices and demands under three distinct compatibility regimes

Incompatibility (NN)

If either application A_j is not compatible with rival hardware H_i, then we are in an incompatible regime NN. First, note that using the advertising demand function (defined by Equations (C.1)), we have

$$s_{Hi} = \frac{(1-z)qD_i + q(N_i - D_i)}{N_i}, s_{Ai} = zq, \text{ and } a_{Hi} = a_{Ai} = 1, \; i \in \{1,2\}. \tag{C.4}$$

Second, since firm i optimally chooses $r_i \in (0, W(Z)]$, using the application demand function (defined by Equation 10), we have $X_i = D_i$. Now, using these values in Equation (1), the profit of firm $i \in \{1,2\}$ is

$$\pi_i \;=\; (p_i + q)N_i + (r_i + zq)D_i - zqD_i. \tag{C.5}$$

Substituting demands from Equations (A.3) and (A.4) in the preceding profit functions, the

first-order conditions to firm i's profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{2} + \frac{p_j - p_i}{2t} + \alpha\frac{(r_j - r_i)}{2t} + (p_i + q)\left(\frac{-1}{2t}\right) + r_i\left(\frac{-\alpha}{2t}\right) \leq 0, \text{ and} \tag{C.6}$$

$$\frac{\partial \pi_i}{\partial r_i} = (p_i + q)\left(\frac{-\alpha}{2t}\right) + \frac{\alpha}{2} + \alpha\frac{(p_j - p_i)}{2t} + \alpha\frac{r_j - r_i}{2t} + r_i\left(\frac{-\alpha}{2t}\right) \leq 0, \tag{C.7}$$

where subscript $j \neq i$ denotes the rival firm. Using Equations (C.6) and (C.7), setting $\frac{\partial \pi_i}{\partial p_i} = 0$, and $\frac{\partial \pi_i}{\partial r_i} = 0$, and imposing symmetry (since firms have symmetric objective functions), we obtain the symmetric optimal hardware prices as

$$p_1^{NN} = p_2^{NN} = t - q, \tag{C.8}$$

which are non-negative, given the assumption of non-negative prices (Assumption 2), and symmetric optimal application prices are

$$r_1^{NN} = r_2^{NN} = 0. \tag{C.9}$$

Since each firmâ€™s profit function π_i is strictly concave in its own choice variables: p_i and r_i (holding the rivalâ€™s prices fixed), and also, the symmetric solution to the firstâ€‘order conditions is strictly interior (i.e., positive), given the assumption of non-negative prices (Assumption 2), the interior solution constitutes the unique global maximum. The equilibrium demand for hardware H_i is $N_i = \frac{1}{2}, i \in \{1,2\}$, the equilibrium demand for application A_i is $X_i = D_i = \frac{\alpha}{2},\ i \in \{1,2\}$, and the equilibrium advertising level is $a_{H1}^{NN} = a_{H2}^{NN} = a_{A1}^{NN} = a_{A2}^{NN} = 1$. Combining this with Equation (C.4), we have

$$s_{H1}^{NN} = s_{H2}^{NN} = q - \alpha z q, \text{ and } s_{A1}^{NN} = s_{A2}^{NN} = zq. \tag{C.10}$$

Using Equation (C.5), firms' profit are $\pi_1^{NN} = \pi_2^{NN} = \frac{t}{2}$.

Full compatibility (CC)

If application A_j is compatible with rival hardware H_i, then we are in full compatibility regime CC. First, note that using the advertising demand function (defined by Equations (C.2)), we have

$$s_{Hi} = \frac{(1-2z)qD_i + q(N_i - D_i)}{N_i}, s_{Ai} = 2zq, \text{ and } a_{Hi} = a_{Ai} = 1, \ i \in \{1,2\}. \tag{C.11}$$

Second, since firm i optimally chooses $r_i \in (0, W(Z)]$, using the application demand function (defined by Equation 13), we have $X_i = D_i + D_j = \alpha$. Now, using these values in Equation (1), the profit of firm $i \in \{1,2\}$ is

$$\pi_i = (p_i + q)N_i + (r_i + 2zq)\alpha - 2zqD_i - F. \tag{C.12}$$

Substituting demands from Equations (A.6) and (A.7) in the preceding profit functions, the first-order conditions to firm i's profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{2} + \frac{p_j - p_i}{2t} + (p_i + q)\left(\frac{-1}{2t}\right) - 2zq\alpha\left(\frac{-1}{2t}\right) \leq 0, \text{ and} \tag{C.13}$$

$$\frac{\partial \pi_i}{\partial r_i} = \alpha > 0. \tag{C.14}$$

Using Equations (C.13), setting $\frac{\partial \pi_i}{\partial p_i} = 0$, and imposing symmetry (since firms have symmetric objective functions), we obtain the symmetric optimal hardware prices as

$$p_1^{CC} = p_2^{CC} = t - q + 2\alpha zq, \tag{C.15}$$

which are non-negative, given the assumption of non-negative prices (Assumption 2). Since each firmâ€™s profit function π_i is strictly concave in its own choice variable: p_i (holding the rivalâ€™s price fixed), and also, the symmetric solution to the firstâ€‘order conditions is strictly interior (i.e., positive), given the assumption of non-negative prices (Assumption 2), the interior solution constitutes the unique global maximum. Next, Equation (C.14) shows that the profits are strictly increasing in application prices. Thus, firm $i \in \{1,2\}$ sets r_i, so that the multi-product user is left with zero surplus from consuming the application. This gives

$$r_1^{CC} = r_2^{CC} = W(z). \tag{C.16}$$

The equilibrium demand for hardware H_i is $N_i = \frac{1}{2}, i \in \{1,2\}$, the equilibrium demand for application A_i is $X_i = D_i = \alpha,\ i \in \{1,2\}$, and the equilibrium demand for advertising is $a_{H1}^{CC} = a_{H2}^{CC} = a_{A1}^{CC} = a_{A2}^{CC} = 1$. Combining this with Equation (C.11), we have

$$s_{H1}^{NN} = s_{H2}^{NN} = q - 2\alpha zq \ \ and \ \ s_{A1}^{NN} = s_{A2}^{NN} = 2zq. \tag{C.17}$$

Using Equation (C.12), firms' profit are $\pi_1^{CC} = \pi_2^{CC} = \frac{t}{2} + \alpha W(z) + 2\alpha zq - F$.

Asymmetric compatibility (NC)

Without loss of generality, suppose that firm 1 chooses to make its application A_1 incompatible with rival firmâ€™s hardware H_2, whereas firm 2 chooses to make its application A_2 compatible with rival firmâ€™s hardware H_1. First, note that using the advertising demand function (defined by Equations (C.3)), we have

$$\begin{aligned} s_{H1} &= \frac{(1-2z)qD_1 + q(N_1 - D_1)}{N_1},\ s_{H2} = \frac{(1-z)qD_2 + q(N_2 - D_2)}{N_2}, \\ s_{A1} &= zq,\ s_{A2} = 2zq,\ \ and \ \ a_{H1} = a_{H2} = a_{A1} = a_{A2} = 1. \end{aligned} \tag{C.18}$$

Second, since firm i optimally chooses $r_i \in (0, W(Z)]$, using the application demand function (defined by Equation 13), $X_1 = D_1$ and $X_2 = D_1 + D_2 = \alpha$. Now, using these values in Equation (1), the profit of firm $i \in \{1,2\}$ is

$$\pi_1 = (p_1 + q)N_1 + (r_1 - zq)D_1 \text{ and } \pi_2 = (p_2 + q)N_2 + (r_2 + 2zq)\alpha - zqD_2 - F. \tag{C.19}$$

Substituting demand from Equations (A.9) and (A.10) in the preceding profit functions, the first-order conditions to firm 1 and firm 2's profit maximization are

$$\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W(z)}{2t} - \frac{\alpha z V}{2t} + (p_1 + q)\left(\frac{-1}{2t}\right) + (r_1 - zq)\left(\frac{-\alpha}{2t}\right) \leq 0, \tag{C.20}$$

$$\frac{\partial \pi_1}{\partial r_1} = (p_1 + q)\left(\frac{-\alpha}{2t}\right) + \frac{\alpha}{2} + \alpha\frac{(p_2 - p_1)}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W(z)}{2t} - \frac{\alpha z V}{2t} + (r_1 - zq)\left(\frac{-\alpha}{2t}\right) \leq 0, \tag{C.21}$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha r_1}{2t} - \frac{\alpha W}{2t} + \frac{\alpha z V}{2t} + (p_2 + q)\left(\frac{-1}{2t}\right) + 0 - zq\left(\frac{-\alpha}{2t}\right) \leq 0, \text{ and} \tag{C.22}$$

$$\frac{\partial \pi_2}{\partial r_2} = \alpha > 0. \tag{C.23}$$

Using first-order conditions defined by Equations (C.20), (C.21) and (C.22), setting $\frac{\partial \pi_1}{\partial p_1} = 0$, $\frac{\partial \pi_1}{\partial r_1} = 0$, and $\frac{\partial \pi_2}{\partial p_2} = 0$, and solving, we obtain the optimal hardware H_1 price as

$$p_1^{NC} = t - q - \frac{\alpha W(z)}{6} + \frac{\alpha z V}{6} + \frac{3\alpha z q}{6}, \tag{C.24}$$

and the optimal hardware H_1 price as

$$p_2^{NC} = t - q - \frac{\alpha W(z)}{3} + \frac{\alpha z V}{3} + \frac{3\alpha z q}{3}, \tag{C.25}$$

which are non-negative, given the assumption of full market coverage (Assumption 1) and non-negative prices (Assumption 2). The optimal application A_1 price is

$$r_1^{NC} = \frac{W(z)}{2} - \frac{zV}{2} + \frac{zq}{2}. \tag{C.26}$$

Since firm 1's (respectively, firm 2's) profit function π_1 (respectively, π_2) is strictly concave in its own choice variables: p_1 and r_1 (respectively, p_2) (holding the rivalâ€™s prices fixed), and also, the solution to the firstâ€‘order conditions is strictly interior (i.e., positive), given the assumption of non-negative prices (Assumption 2), the interior solution constitutes the unique global maximum.

Equation (C.23) shows that the profit of firm 2 is strictly increasing in application price r_2. Thus, firm 2 sets r_2 to extract as much consumer surplus as possible, while also ensuring that all multi-product users consume application A_2, i.e., she obtains a non-negative payoff from consuming application A_2. Using the application demand function (defined by Equation (16)), we have

$$r_2^{NC} = W(z). \tag{C.27}$$

The equilibrium demand is obtained by substituting Equations (C.24), (C.25), and (C.26) in Equations (A.9) and (A.10) to get

$$N_1^{NC} = \frac{1}{2} + \frac{\alpha W(z)}{6t} - \frac{\alpha z V}{6t},\ N_2^{NC} = 1 - N_2^{NC};$$
$$X_1^{NC} = D_1^{NC} = \alpha\left[\frac{1}{2} + \frac{(3-\alpha)W(z)}{12t} - \frac{(3-\alpha)zV}{12t} - \frac{(3-3\alpha)zq}{12t}\right],\ \text{and } X_2^{NC} = D_1 + D_2 = \alpha,$$

and the equilibrium advertising level is $a_{H1}^{NC} = a_{H2}^{NC} = a_{A1}^{NC} = a_{A2}^{NC} = 1$. Combining this with Equation (C.18), we have

$$s_{H1}^{NC} = \frac{(1-2z)qD_1^{NC} + q(N_1^{NC} - D_1^{NC})}{N_1^{NC}},\ s_{H2} = \frac{(1-z)qD_2^{NC} + q(N_2^{NC} - D_2^{NC})}{N_2^{NC}},\ s_{A1}^{NC} = zq,\ and\ s_{A2}^{NC} = 2zq. \tag{C.28}$$

Using Equation (C.19), firms' profit are $\pi_1^{NC} = (p_1^{NC} + q)N_1^{NC} + (r_1^{NC} - 2zq)D_1^{NC}$, and $\pi_2^{NC} = (p_2^{NC} + q)N_2^{NC} + r_2^{NC}(D_1^{NC} + D_2^{NC}) - zqD_2^{NC} - F$.

Combining Equations (C.8), (C.15), (C.24), and (C.25), we have

$$p_2^{NC} \leq p_1^{NC} \leq p_1^{NN} = p_2^{NN} \leq p_1^{CC} = p_2^{CC}, \tag{C.29}$$

where the final inequality holds for all values of $\alpha \in [0,1]$, given the assumption of non-negative prices (Assumption 2).

Combining Equations (C.9), (C.16), (C.26) and (C.27), we have

$$r_2^{NN} = r_1^{NN} \leq r_1^{NC} \leq r_2^{NC} = r_2^{CC} = r_1^{CC},$$

given the assumption of full market coverage (Assumption 1).

Combining Equations (C.10), (C.17), (C.28), and using Claims 2 and 3, we can show that

$$s_{H1}^{NC} < s_{H1}^{CC} = s_{H2}^{CC} < s_{H1}^{NN} = s_{H2}^{NN} < s_{H2}^{NC},$$

and

$$s_{A1}^{NC} = s_{A1}^{NN} = s_{A2}^{NN} < s_{A2}^{CN} = s_{A1}^{CC} = s_{A2}^{CC}.$$

This completes the comparison of prices across compatibility regimes.[C.1]

C.3 Stage 1: Equilibrium compatibility regime

Proof of Proposition 4

We proceed through a series of steps. In *Step 1*, we characterize the conditions for the adoption of compatibility by the firms. In *Step 2*, we compare the thresholds obtained from *Step 1*. In *Step 3*, we characterize firms' optimal strategies. In *Step 4*, we compare the equilibrium outcomes with those under baseline model.

Step 1: Conditions for choosing compatibility by the firms and comparison of thresholds.

Suppose firm $i \in \{1, 2\}$ believes that firm $j \neq i$ chooses incompatibility. Then, firm i is indifferent between choosing incompatibility and compatibility when compatibility cost is $F_1(\alpha)$, such that (i) $\pi_i^{NN}(F) = \pi_i^{CN}(F)$, at $F = F_1(\alpha)$, (ii) $\pi_i^{NN}(F) < \pi_i^{CN}(F)$, for all $F < F_1(\alpha)$, and (iii) $\pi_i^{NN}(F) > \pi_i^{CN}(F)$, for all $F > F_1(\alpha)$, where $F_1(\alpha)$ is obtained by solving

$$\begin{aligned}\frac{t}{2} = &\left[t - \frac{\alpha W(z)}{3} + \frac{\alpha z V}{3} + \frac{3\alpha z q}{3}\right] \cdot \left[\frac{1}{2} - \frac{\alpha W(z)}{6t} + \frac{\alpha z V}{6t}\right] + \alpha W(z) + 2\alpha z q \\ &- \alpha z q \left[\frac{1}{2} - \frac{(3-\alpha)W(z)}{12t} + \frac{(3-\alpha)zV}{12t} + \frac{(3-3\alpha)zq}{6t}\right] - F_1(\alpha),\end{aligned}$$

[C.1]Following the same argument, under asymmetric compatibility regime CN, we can show that $p_1^{CN} \leq p_2^{CN} \leq p_1^{NN} = p_2^{NN} \leq p_1^{CC} = p_2^{CC}$, $r_2^{NN} = r_1^{NN} \leq r_2^{CN} \leq r_1^{CN} = r_2^{CC} = r_1^{CC}$, $s_{H2}^{CN} < s_{H1}^{CC} = s_{H2}^{CC} < s_{H1}^{NN} = s_{H2}^{NN} < s_{H1}^{CN}$, and $s_{A2}^{CN} = s_{A1}^{NN} = s_{A2}^{NN} < s_{A1}^{CN} = s_{A1}^{CC} = s_{A2}^{CC}$.

$$F_1(\alpha) = \left[t - \frac{\alpha W(z)}{3} + \frac{\alpha z V}{3} + \frac{3\alpha z q}{3}\right] \cdot \left[\frac{1}{2} - \frac{\alpha W(z)}{6t} + \frac{\alpha z V}{6t}\right] + \alpha W(z) + 2\alpha z q$$
$$- \alpha z q \left[\frac{1}{2} - \frac{(3-\alpha)W(z)}{12t} + \frac{(3-\alpha)zV}{12t} + \frac{(3-3\alpha)zq}{6t}\right] - \frac{t}{2}. \tag{C.30}$$

In summary, $F_1(\alpha)$ is the threshold for compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application incompatible.

Suppose that firm $i \in \{1, 2\}$ believes that firm $j \neq i$ chooses compatibility. Then, firm i is indifferent between choosing incompatibility and compatibility when its compatibility cost is $F_2(\alpha)$, such that (i) $\pi_i^{NC}(F) = \pi_i^{CC}(F)$, at $F = F_2(\alpha)$, (ii) $\pi_i^{NC}(F) < \pi_i^{CC}(F)$, for all $F < F_2(\alpha)$, and (iii) $\pi_i^{NC}(F) > \pi_i^{CC}(F)$, for all $F > F_2(\alpha)$, where $F_2(\alpha)$ is obtained by solving

$$\frac{t}{2} + \alpha W(z) + 2\alpha z q - F_2(\alpha) = \left[t - \frac{\alpha W(z)}{6} + \frac{\alpha z V}{6} + \frac{3\alpha z q}{6}\right] \cdot \left[\frac{1}{2} + \frac{\alpha W(z)}{6t} - \frac{\alpha z V}{6t}\right]$$
$$+ \alpha \left[\frac{W(z)}{2} - \frac{zV}{2} - \frac{zq}{2}\right] \cdot \left[\frac{1}{2} + \frac{(3-\alpha)W(z)}{12t} - \frac{(3-\alpha)zV}{12t} - \frac{(3-3\alpha)zq}{6t}\right],$$

$$F_2(\alpha) = \frac{t}{2} + \alpha W(z) + 2\alpha z q - \left[t - \frac{\alpha W(z)}{6} + \frac{\alpha z V}{6} + \frac{3\alpha z q}{6}\right] \cdot \left[\frac{1}{2} + \frac{\alpha W(z)}{6t} - \frac{\alpha z V}{6t}\right]$$
$$- \alpha \left[\frac{W(z)}{2} - \frac{zV}{2} - \frac{zq}{2}\right] \cdot \left[\frac{1}{2} + \frac{(3-\alpha)W(z)}{12t} - \frac{(3-\alpha)zV}{12t} - \frac{(3-3\alpha)zq}{6t}\right]. \tag{C.31}$$

In summary, $F_2(\alpha)$ is the threshold for compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application compatible.

Step 2: Comparison of thresholds $F_1(\alpha)$ and $F_2(\alpha)$.

Claim 10. $\frac{\partial F_1(\alpha)}{\partial \alpha} > 0$, *for all* $\alpha > 0$.

Proof. Using Equation (C.30), and differentiating $F_1(\alpha)$ with respect to α gives

$$\begin{aligned}\frac{\partial F_1(\alpha)}{\partial \alpha} &= -\left[\frac{W(z)}{3} - \frac{zV}{3} - \frac{3zq}{3}\right] \cdot N_i^{CN} - \left[t - \frac{\alpha W(z)}{3} + \frac{\alpha zV}{3} + \frac{3\alpha zq}{3}\right] \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t}\right] \\ &+ W(z) + 2zq - zq \cdot \left[\frac{1}{2} - \frac{(3-2\alpha)W(z)}{12t} + \frac{(3-2\alpha)zV}{12t} + \frac{(3-6\alpha)zq}{12t}\right].\end{aligned} \tag{C.32}$$

Let us define

$$B := W(z) + 2zq - zq \cdot \left[\frac{1}{2} - \frac{(3-2\alpha)W(z)}{12t} + \frac{(3-2\alpha)zV}{12t} + \frac{(3-6\alpha)zq}{12t}\right].$$

Note that $N_i^{CN} \leq \frac{1}{2}$ and $p_i^{CN} \leq t$. Using these inequalities and the expression for $\frac{\partial F_1(\alpha)}{\partial \alpha}$, we obtain

$$\begin{aligned}\frac{\partial F_1(\alpha)}{\partial \alpha} &\geq B - \frac{1}{2} \cdot \left[\frac{W(z)}{3} - \frac{zV}{3} - \frac{3zq}{3}\right] - t \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t}\right] \\ &= B - \frac{1}{6} \cdot \left[2W(z) - 2zV - 3zq\right] \\ &= \frac{2W(z)}{3} + \frac{zV}{3} + 2zq + zq \cdot \left[\frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-6\alpha)2zq}{12t}\right] + 2zq \;>\; 0,\end{aligned}$$

where the final inequality holds for all $\alpha > 0$, given the assumption of full market coverage (Assumption 1). Hence, we have $\frac{\partial F_1(\alpha)}{\partial \alpha} > 0$, for all $\alpha > 0$. ■

Claim 11. $\frac{\partial F_2(\alpha)}{\partial \alpha} > 0$, *for all* $\alpha > 0$.

Proof. Using Equation (C.31), and differentiating $F_2(\alpha)$ with respect to α gives

$$\begin{aligned}\frac{\partial F_2(\alpha)}{\partial \alpha} &= W(z) + 2zq - \left[-\frac{W(z)}{6} + \frac{zV}{6} + \frac{3zq}{6}\right] \cdot N_i^{NC} \\ &- \left[t - \frac{\alpha W(z)}{6} + \frac{\alpha zV}{6} + \frac{3\alpha zq}{6}\right] \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t}\right] \\ &- \left[\frac{W(z)}{2} - \frac{zV}{2} - \frac{zq}{2}\right] \cdot \left[\frac{1}{2} + \frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-6\alpha)zq}{12t}\right].\end{aligned} \tag{C.33}$$

Let us define

$$B := \left[\frac{W(z)}{2} - \frac{zV}{2} - \frac{zq}{2}\right].$$

Note that $N_i^{NC} \geq \frac{1}{2}$ and $p_i^{NC} \leq t$. Using these inequalities and the expression for $\frac{\partial F_2(\alpha)}{\partial \alpha}$, we obtain

$$\begin{aligned}\frac{\partial F_2(\alpha)}{\partial \alpha} &> W(z) + 2zq + \frac{1}{2} \cdot \left[\frac{W(z)}{6} - \frac{zV}{6} - \frac{3zq}{6}\right] - \left[\frac{W(z)}{6} - \frac{zV}{6}\right] \\ &\quad - \frac{1}{2} \cdot B - B\left[\frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-6\alpha)zq}{12t}\right] \\ &= \frac{2W(z)}{3} + \frac{zV}{3} + 2zq - B\left[\frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-6\alpha)zq}{12t}\right] \\ &> \frac{2W(z)}{3} + \frac{zV}{3} + 2zq - B\left[\frac{W(z)}{4t} - \frac{zV}{4t} - \frac{zq}{4t}\right] \\ &= \frac{2W(z)}{3} + \frac{zV}{3} + 2zq - \left[\frac{W(z)}{2} - \frac{zV}{2} - \frac{zq}{2}\right]\left[\frac{W(z)}{4t} - \frac{zV}{4t} - \frac{zq}{4t}\right] \\ &= \frac{2W(z)}{3} + \frac{zV}{3} + 2zq - \frac{1}{8t}[W(z) - zV - zq]^2 > 0,\end{aligned}$$

where the second-last inequality follows because $\left[\frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-4\alpha)2zq}{12t}\right]$ is strictly decreasing in α, and last inequality follows because $t > \frac{3[W(z)-zV-zq]^2}{8[2W(z)+zV+2zq]}$, given the assumption of non-negative prices (Assumption 2). ■

Claim 12. *$\frac{\partial F_1(\alpha)}{\partial \alpha} > \frac{\partial F_2(\alpha)}{\partial \alpha}$, for all $\alpha > 0$.*

Proof. Note that since firms are symmetric, $N_i^{CN} = 1 - N_i^{NC}$. Also, $N_i^{NC} > \frac{1}{2}$, and $\bar{x} > \frac{1}{2}$ (where $\bar{x}$ is defined by Equation (A.8)). Combining this with Equations (B.31) and (B.32), we have

$$\begin{aligned}\frac{\partial F_1(\alpha)}{\partial \alpha} - \frac{\partial F_2(\alpha)}{\partial \alpha} &= \left[\frac{W(z)}{6} - \frac{zV}{6} - \frac{zq}{2}\right] \cdot N_i^{NC} - \left[\frac{W(z)}{3} - \frac{zV}{3}\right] \\ &\quad + \left[\frac{\alpha W(z)}{6} - \frac{\alpha zV}{6} - \frac{3\alpha zq}{6}\right] \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t}\right] \\ &\quad + \left[\frac{W(z)}{2} - \frac{zV}{2} + \frac{zq}{2}\right] \cdot \left[\frac{1}{2} + \frac{(3-2\alpha)W(z)}{12t} - \frac{(3-2\alpha)zV}{12t} - \frac{(3-6\alpha)zq}{12t}\right] \\ &> \left[\frac{W(z)}{6} - \frac{zV}{6} - \frac{zq}{2}\right] \cdot \frac{1}{2} + \left[\frac{W(z)}{2} - \frac{zV}{2} + \frac{zq}{2}\right] \cdot \frac{1}{2} - \left[\frac{W(z)}{3} - \frac{zV}{3}\right] \\ &\quad + \left[\frac{\alpha W(z)}{6} - \frac{\alpha zV}{6} - \frac{3\alpha zq}{6}\right] \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t}\right] \\ &= \left[\frac{\alpha W(z)}{6} - \frac{\alpha zV}{6} - \frac{3\alpha zq}{6}\right] \cdot \left[\frac{W(z)}{6t} - \frac{zV}{6t}\right] + \frac{zq}{4} \geq 0,\end{aligned}$$

where the final inequality holds for all values of $\alpha \geq 0$, given the assumption of full market coverage (Assumption 1). ■

Finally, note that, at $\alpha = 0$, the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ take the same values, $F_1(0) = F_2(0) = 0$. Combining this with Claims 10, 11, and 12, we have $F_1(\alpha) \geq F_2(\alpha)$ for all $\alpha \in [0, 1]$.

Step 3: Optimal strategies and equilibrium outcomes.

Since, $F_1(\alpha) \geq F_2(\alpha)$ for all $\alpha \in [0, 1]$, in a market characterized by compatibility cost (F) and a fraction of multi-product users (α), the following holds.

(i) For a sufficiently small compatibility cost, i.e., $F < F_2(\alpha)$, both firms choose compatibility, i.e., regime CC, is the equilibrium outcome. Whereas, for a sufficiently large compatibility cost, i.e., $F \geq F_1(\alpha)$, both firms choose incompatibility, i.e., regime NN, is the equilibrium outcome.

(ii) Otherwise, there exist multiple equilibria with either firm 1 choosing incompatibility and firm 2 choosing compatibility, i.e., regime NC, or firm 1 choosing compatibility and firm 2 choosing incompatibility, i.e., regime CN, as the equilibrium outcome.

Step 4: Comparison with the baseline model.

Now, we compare the thresholds $F_1(\alpha)$ (defined by Equation (C.30)) and $F_2(\alpha)$ (defined by Equation (C.31)) on compatibility costs with the thresholds obtained under the baseline model. For notational clarity, let us denote thresholds on compatibility costs under the baseline model as $F_1^h(\alpha)$ (defined by Equation (B.29)) and $F_2^h(\alpha)$ (defined by Equation (B.30)). Therefore, using Equations (B.29), (B.30), (C.30), and (C.31), and comparing the thresholds shows that

$$F_1^h(\alpha) \;\leq\; F_1(\alpha) \quad \textit{and} \quad F_2^h(\alpha) \leq F_2(\alpha).$$

This shows that the region, $F \geq F_1(\alpha)$, where incompatibility regime NN is the equilibrium outcome, is smaller than that under the baseline model. Similarly, the region, $F < F_2(\alpha)$, where full compatibility regime CC is the equilibrium outcome, is larger than that under the baseline model. This completes the proof. ■

D Endogenous Hardware Multi-Homing

In this extension, we allow users to choose whether to consume one hardware device (single-home) or both hardware devices (multi-home). As discussed in Section 5.2, a single-product user who consumes both hardware devices (multi-home) spends λ (respectively, $1-\lambda$) unit of time on hardware H_1 (respectively, hardware H_2). A multi-product user who consumes both hardware devices (multi-home) always spends z unit of time on each application, $\lambda(1-2z)$ (respectively, $1-\lambda(1-2z)$) unit of time on hardware H_1 (respectively, hardware H_2).

D.1 Stages 5, 4 and 3: Demand from single-product and multi-product users, and advertising demand functions

If a single-product user located at $x \in [0,1]$ purchases hardware H_i at price p_i, then her net utility is

$$U_i(x) = \begin{cases} V - p_1 - tx, & \text{if she consumes } H_1 \ (i=1), \\ V - p_2 - t(1-x), & \text{if she consumes } H_2 \ (i=2), \text{ and} \\ \lambda V + (1-\lambda)V - p_2 - t, & \text{if she consumes both } H_1 \text{ and } H_2 \ (i=12). \end{cases} \tag{D.1}$$

Claim 13. *Single-product users prefer consuming only one hardware device (single-homing) over consuming both hardware devices (multi-homing).*

Proof: Using Equation (D.1), we have $U_1(x) \geq U_{12}(x)$, and $U_2(x) \geq U_{12}(x)$ for all $x \in [0,1]$, and $p_i \geq 0$, $i \in \{1,2\}$, with strict inequality for $p_i > 0$. Therefore, a single-product user located at $x \in [0,1]$, prefers consuming only one hardware device (single-homing) over consuming both hardware devices (multi-homing). ■

Therefore, the location of single-product user indifferent between single-homing with hardware H_1 and H_2 is as defined by Equation (A.1). Thus, demand functions remain the same as under the baseline model.

Next, depending on the compatibility regime, we can have three different scenarios for multi-product users.

Incompatibility (NN)

If application A_j is not compatible with rival hardware H_i, where $j \neq i$, then we are in an incompatible regime NN. At *Stage 5*, we obtain the demand X_i for application A_i as defined by Equation (10). At *Stage 4*, a multi-product user's net utility is

$$U_i(x) = \begin{cases} (1-z)V - p_1 + W(z) - r_1 - tx, & \text{if she consumes } H_1 \text{ and } A_1\ (i=1), \\ (1-z)V - p_2 + W(z) - r_2 - t(1-x), & \text{if she consumes } H_2 \text{ and } A_2\ (i=2), \text{ and} \\ (1-2z)[\lambda V + (1-\lambda)V] - p_1 - p_2 + 2W(z) - r_1 - r_2 - t, & \text{if she consumes } H1, H_2, A1, \\ & \text{and } A_2\ (i=12). \end{cases} \tag{D.2}$$

Using Equation (D.2), the multi-product user indifferent between single-homing with hardware H_1 and multi-homing is located at $\bar{x}_m$, such that (i) $U_1(\bar{x}_m) = U_{12}(\bar{x}_m)$, (ii) $U_1(x) > U_{12}(x)$, for all $x < \bar{x}_m$, and (iii) $U_1(x) < U_{12}(x)$, for all $x > \bar{x}_m$, where

$$\bar{x}_m = 1 - \frac{(W(z) - zV)}{t} + \frac{p_2}{t} + \frac{r_2}{t}. \tag{D.3}$$

Using Equation (D.2), the multi-product user indifferent between single-homing with hardware H_2 and mutli-homing is located at $\bar{x}_M$, such that (i) $U_2(\bar{x}_M) = U_{12}(\bar{x}_M)$, (ii) $U_2(x) < U_{12}(x)$, for all $x < \bar{x}_M$, and (iii) $U_2(x) > U_{12}(x)$, for all $x > \bar{x}_M$, where

$$\bar{x}_M = \frac{W(z) - zV}{t} - \frac{p_1}{t} - \frac{r_1}{t}. \tag{D.4}$$

This gives number of single-homing multi-product users on firms 1 and 2 as

$$D_1 = \bar{x}_m = 1 - \frac{(W(z) - zV)}{t} + \frac{p_2}{t} + \frac{r_2}{t}, \quad D_2 = 1 - \bar{x}_M = 1 - \frac{(W(z) - zV)}{t} + \frac{p_1}{t} + \frac{r_1}{t}, \tag{D.5}$$

and the number of multi-homing multi-product users as

$$D_{12} = \bar{x}_M - \bar{x}_m = 2\frac{(W(z) - zV)}{t} - \frac{p_1}{t} - \frac{p_2}{t} - \frac{r_1}{t} - \frac{r_2}{t} - 1. \tag{D.6}$$

Combining Equations (A.1), (D.3), and (D.4) gives the total demand for hardware H_1 and H_2

(from both single-product and multi-product users) as

$$N_1 = (1-\alpha)\hat{x} + \alpha\bar{x}_m + \alpha(\bar{x}_M - \bar{x}_m) = (1-\alpha)\left[\frac{1}{2} + \frac{p_2 - p_1}{2t}\right] + \alpha\left[\frac{W(z) - zV}{t} - \frac{p_1}{t} - \frac{r_1}{t}\right], \text{ and}$$

$$N_2 = (1-\alpha)(1-\hat{x}) + \alpha(1-\bar{x}_M) + \alpha(\bar{x}_M - \bar{x}_m) = (1-\alpha)\left[\frac{1}{2} + \frac{p_2 - p_1}{2t}\right] + \alpha\left[\frac{W(z) - zV}{t} - \frac{p_2}{t} - \frac{r_2}{t}\right]. \tag{D.7}$$

At *Stage 3*, given advertising prices $s_i, i \in \{1,2\}$, an advertiser places an advertisement in hardware $H_i, i \in \{1,2\}$ if $(1-z)qD_i + (1-2z)qD_{12} + q(N_i - D_i - D_{12}) - s_iN_i \geq 0$. This gives the level of advertisements a_i in hardware $H_i, i \in \{1,2\}$ as

$$a_i = \begin{cases} 1, & \text{if } (1-z)qD_i + (1-2z)qD_{12} + q(N_i - D_i - D_{12}) \geq s_iN_i, \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{D.8}$$

Full compatibility (CC)

If application A_j is compatible with rival hardware H_i, where $j \neq i$, then we are in full compatibility regime CC. At *Stage 5*, we obtain the demand X_i for application A_i as defined by Equation (13). At *Stage 4*, a multi-product user's net utility is

$$U_i(x) = \begin{cases} (1-2z)V - p_1 + 2W(z) - r_1 - r_2 - tx, & \text{if she consumes } H1, A1, \\ & \text{and } A_2\ (i=1) \\ (1-2z)V - p_2 + 2W(z) - r_1 - r_2 - t(1-x), & \text{if she consumes } H2, A1, \\ & \text{and } A_2\ (i=2) \\ (1-2z)[\lambda V + (1-\lambda)V] - p_1 - p_2 + 2W(z) - r_1 - r_2 - t, & \text{if she consumes } H1, H_2, A1, \\ & \text{and } A_2\ (i=12). \end{cases} \tag{D.9}$$

Claim 14. *Under full compatibility regime CC, multi-product users prefer consuming only one hardware device (single-homing) over consuming both hardware devices (multi-homing).*

Proof: Using Equation (D.9), we have $U_1(x) \geq U_{12}(x)$, and $U_2(x) \geq U_{12}(x)$ for all $x \in [0,1]$, and

$p_i \geq 0$, $i \in \{1, 2\}$, with strict inequality for $p_i > 0$. Therefore, a multi-product user located at $x \in [0, 1]$, prefers consuming only one hardware device (single-homing) over consuming both hardware devices (multi-homing). ■

Therefore, the multi-product user indifferent between single-homing with hardware H_1 and H_2, is located at $\bar{x}$ as defined by Equation (A.5) under the baseline model. This yields demand functions as under the baseline model (defined by Equation (A.6)). Moreover, the total demand functions for hardware H_1 and H_2 from both single-product and multi-product users remain the same as under baseline model (defined by Equation (A.7)).

At *Stage 3*, given advertising price $s_i, i \in \{1, 2\}$, an advertiser places an advertisement in hardware as defined by Equation (15).

Asymmetric compatibility (NC)

Without loss of generality, suppose that application A_1 is incompatible with hardware H_2, whereas application A_2 is compatible with hardware H_1. Then we are in an asymmetric compatibility regime NC. At *Stage 5*, we obtain the demand X_i for application A_i as defined by Equation (16). At *Stage 4*, a multi-product user's net utility is

$$U_i(x) = \begin{cases} (1-2z)V - p_1 + 2W(z) - r_1 - r_2 - tx, & \text{if she consumes } H_1, A_1, \\ & \text{and } A_2\ (i=1), \\ (1-z)V - p_2 + W(z) - r_2 - t(1-x), & \text{if she consumes } H_2, \\ & \text{and } A_2\ (i=2), \text{ and} \\ (1-2z)V[\lambda V + (1-\lambda)V] - p_1 - p_2 + 2W(z) - r_1 - r_2 - t(1-x), & \text{if she consumes } H1, H_2, A1, \\ & \text{and } A_2\ (i=12). \end{cases} \tag{D.10}$$

Claim 15. *Under asymmetric compatibility regime NC, multi-product users prefer consuming hardware device H_1 (single-homing) over consuming both hardware devices (multi-homing).*

Proof: Using Equation (D.10), we have $U_1(x) \geq U_{12}(x)$, for all $x \in [0, 1]$, and $p_i, r_i \geq 0$, $i \in \{1, 2\}$, with strict inequality for $p_i > 0$ or $r_i > 0$. Therefore, a multi-product user located at $x \in [0, 1]$, prefers consuming only hardware H_1 (single-homing) over consuming both hardware devices (multi-homing). Thus, she will never multi-home. ■

Therefore, the multi-product user indifferent between single-homing with hardware H_1 and H_2, is located at $\bar{x}$ as defined by Equation (A.8) under the baseline model. This yields demand functions as under the baseline model (defined by Equation (A.9). Moreover, the total demand functions for hardware H_1 and H_2 from both single-product and multi-product users remain the same as under baseline model (defined by Equation (A.10)).

At *Stage 3*, given advertising price $s_i, i \in \{1, 2\}$, an advertiser places an advertisement in hardware as defined by Equation (18).

D.2 Stage 2: Equilibrium prices and demands under three distinct compatibility regimes

Incompatibility (NN)

If either application A_j is not compatible with rival hardware H_i, then we are in an incompatible regime NN. First, note that using the advertising demand function (defined by Equations (D.8)), we have

$$s_i = \frac{(1-z)qD_i + (1-2z)qD_{12} + q(N_i - D_i - D_{12})}{N_i}, \quad \text{and} \quad a_i = 1, \; i \in \{1, 2\}. \tag{D.11}$$

Second, since firm i optimally chooses $r_i \in (0, W(Z)]$, using the application demand function (defined by Equation 10), we have $X_i = D_i + D_{12}$. Now, using these values in Equation (1), the profit of firm $i \in \{1, 2\}$ is

$$\pi_i \;=\; p_i N_i + r_i(D_i + D_{12}) + q(N_i - zD_i - 2zD_{12}). \tag{D.12}$$

Substituting demands from Equations (D.5) and (D.7) in the preceding profit functions, solving the first-order conditions to firm i's profit maximization, and imposing symmetry (since firms have symmetric objective functions), we obtain the symmetric optimal hardware prices as

$$p_1^{NN} = p_2^{NN} = t - q, \tag{D.13}$$

which are non-negative, given the assumption of non-negative prices (Assumption 2), and symmetric optimal application prices are

$$r_1^{NN} = r_2^{NN} = \frac{1}{2}[(2zq - zV + W(z)) - 2t + q]. \quad \text{(D.14)}$$

Since each firmâ€™s profit function π_i is strictly concave in its own choice variables: p_i and r_i (holding the rivalâ€™s prices fixed), and also, the symmetric solution to the firstâ€‘order conditions is strictly interior (i.e., positive), given the assumption of non-negative prices (Assumption 2), the interior solution constitutes the unique global maximum. Using Equation (D.12), firms' profit are

$$\pi_1^{NN} = \pi_2^{NN} = \frac{t}{2} + \frac{\alpha[-2t^2 + (zV - W(z))^2 + q^2(1 - 6z + 8z^2) + 2q(2t + (-1 + 3z)(zV - W(z)))]}{4t}. \quad \text{(D.15)}$$

For regimes CC and NC, since the total demand functions for each hardware and application and the advertising demand are the same as under the baseline model, the optimal prices, demands and profit in each regimes CC and NC are as characterized in Lemmas 2 and 3, respectively.

D.3 Stage 1: Equilibrium compatibility regime

Proof of Proposition 5

We proceed through a series of steps. In *Step 1*, we characterize the conditions for the adoption of compatibility by the firms. In *Step 2*, we numerically compare the thresholds obtained from *Step 1*. In *Step 3*, we characterize firms' optimal strategies.

Step 1: Conditions for choosing compatibility by the firms and comparison of thresholds.

Suppose firm $i \in \{1, 2\}$ believes that firm $j \neq i$ chooses incompatibility. Then, firm i is indifferent between choosing incompatibility and compatibility when compatibility cost is $F_1(\alpha)$, such that (i) $\pi_i^{NN}(F) = \pi_i^{CN}(F)$, at $F = F_1(\alpha)$, (ii) $\pi_i^{NN}(F) < \pi_i^{CN}(F)$, for all $F < F_1(\alpha)$, and (iii)

$\pi_i^{NN}(F) > \pi_i^{CN}(F)$, for all $F > F_1(\alpha)$, where $F_1(\alpha)$ is obtained by solving

$$F_1(\alpha) = \left[t - \frac{\alpha W(z)}{3} + \frac{\alpha z V}{3} + \frac{4\alpha z q}{3}\right] \cdot \left[\frac{1}{2} - \frac{\alpha W(z)}{6t} + \frac{\alpha z V}{6t} + \frac{\alpha z q}{6t}\right] + \alpha W(z)$$
$$- \alpha z q \left[\frac{1}{2} - \frac{(3-\alpha)W(z)}{12t} + \frac{(3-\alpha)zV}{12t} - \frac{(3-2\alpha)zq}{6t}\right]$$
$$- \frac{t}{2} - \left[\frac{\alpha(-2t^2 + (zV - W(z))^2 + q^2(1 - 6z + 8z^2) + 2q(2t + (-1+3z)(zV - W(z))))}{4t}\right]. \tag{D.16}$$

In summary, $F_1(\alpha)$ is the threshold on compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application incompatible.

Suppose that firm $i \in \{1, 2\}$ believes that firm $j \neq i$ chooses compatibility. Then, firm i is indifferent between choosing incompatibility and compatibility when its compatibility cost is $F_2(\alpha)$, such that (i) $\pi_i^{NC}(F) = \pi_i^{CC}(F)$, at $F = F_2(\alpha)$, (ii) $\pi_i^{NC}(F) < \pi_i^{CC}(F)$, for all $F < F_2(\alpha)$, and (iii) $\pi_i^{NC}(F) > \pi_i^{CC}(F)$, for all $F > F_2(\alpha)$, where $F_2(\alpha)$ is as defined by Equation (B.30). In summary, $F_2(\alpha)$ is the threshold for compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application compatible.

Step 2: Comparison of thresholds $F_1(\alpha)$ and $F_2(\alpha)$.

Due to analytical intractability, we rely on numerical analysis and compare thresholds $F_1(\alpha)$ (defined by Equation (D.16)) and $F_2(\alpha)$ (defined by Equation (B.30)). Using Example 1, we conduct numerical analysis with parameter values: $V = 2$, $w = 6, 8$, $q = 0.7$, $t = 1$, $z = 0.3$ and $b = 0.5$. We find that there can be two distinct scenarios. When users obtain small value from consuming an application ($w = 6$) then the comparison of thresholds coincide with the baseline case, i.e., $F_1(\alpha) \geq F_2(\alpha)$, for all $\alpha \in [0, 1]$, with strict inequality for $\alpha \in (0, 1]$ (refer Figure D.1(i)). Whereas, when users obtain a sufficiently large value from consuming an application ($w = 8$) then, in contrast to the baseline case, we have $F_2(\alpha) \geq F_1(\alpha)$, for all $\alpha \in [0, 1]$, with strict inequality for $\alpha \in (0, 1]$ (refer Figure D.1(ii)).

Step 3: Optimal strategies and equilibrium outcomes.

When users obtain a small value from consuming an application, then from *Step 2*, $F_1(\alpha) \geq$

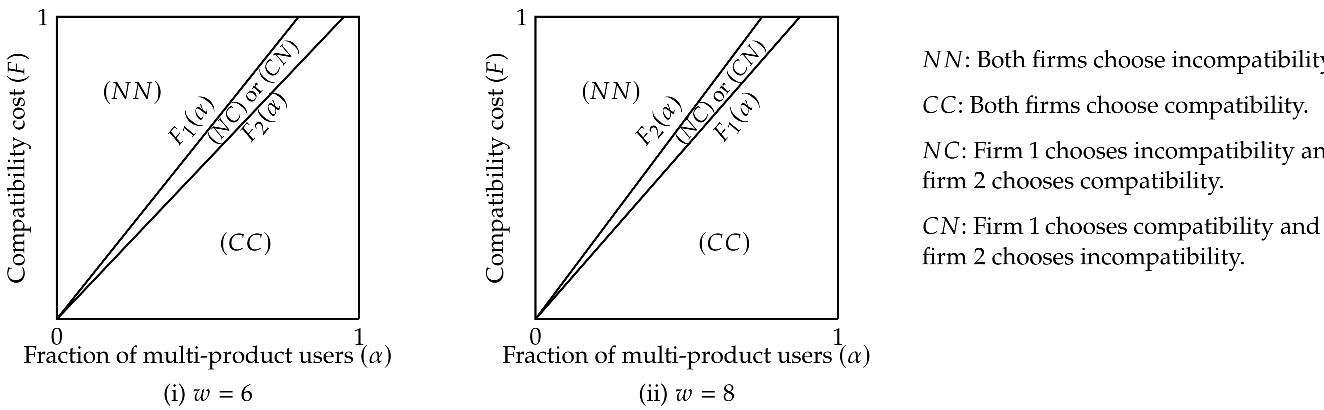


Figure D.1: Optimal compatibility regimes under market equilibrium with multi-homing users

The figure is based on Example 1 with parameter values $V = 2$, $w = 6, 8$, $q = 0.7$, $t = 1$, $z = 0.3$ and $b = 0.5$. The threshold $F_1(\alpha)$ (respectively, $F_2(\alpha)$) represents the loci of points along which a firm is indifferent between choosing compatibility and incompatibility when it believes that rival has chosen incompatibility (respectively, compatibility).

$F_2(\alpha)$ for all $\alpha \in [0, 1]$. Therefore, in a market characterized by compatibility cost (F) and a fraction of multi-product users (α), the equilibrium market regimes is as characterized in Proposition 2. However, when users obtain a sufficiently large value from consuming an application, then from *Step 2*, $F_2(\alpha) \geq F_1(\alpha)$, for all $\alpha \in [0, 1]$. Therefore, in a market characterized by compatibility cost (F) and a fraction of multi-product users (α), the following holds.

(i) For a sufficiently small compatibility cost, i.e., $F < F_1(\alpha)$, both firms choose compatibility, i.e., regime CC, is the equilibrium outcome. Whereas, for a sufficiently large compatibility cost, i.e., $F \geq F_2(\alpha)$, both firms choose incompatibility, i.e., regime NN, is the equilibrium outcome.

(ii) Otherwise, there exist multiple equilibria with either both firms choose incompatibility, i.e., regime NN, or both firms choose compatibility, i.e., regime CC, as the equilibrium outcome. ■

E Endogenous Time Allocation between Hardware and Application

In this extension, we introduce an additional stage at the end of our baseline game, *Stage 6*, at which multi-product users decide how much time to spend on the hardware and the application(s) available on it.

E.1 Stage 6: Users' choice of time allocation between hardware and software

Proof of Lemma 4

Proof: For each of the regimes, we derive users' optimal choice of time allocation between hardware and application, and show that it remains the same across regimes and is independent of the fraction of multi-product users, α.

Incompatibility (NN)

If application A_j is not compatible with rival hardware H_i, where $j \neq i$, then we are in an incompatible regime NN. A multi-product user's utility is

$$U_i(x,z) = \begin{cases} (1-z)V - p_1 + W(z) - r_1 - tx, & \text{if she consumes } H_1 \text{ and } A_1\ (i=1), \text{ and} \\ (1-z)V - p_2 + W(z) - r_2 - t(1-x), & \text{if she consumes } H_2 \text{ and } A_2\ (i=2). \end{cases} \tag{E.1}$$

Differentiating $U_i(x,z)$ (defined by Equation (E.1)) with respect to z, yields a multi-product user's optimal choice z_u, where z_u is characterized by

$$\frac{\partial W(z_u)}{\partial z} = V, \in \{1,2\}. \tag{E.2}$$

Since $W(z)$ is twice continuously differentiable with $W'(z) > 0$, and $W''(z) < 0$, for all $z \in (0,1)$, we have $\frac{\partial^2 U}{\partial z^2} = W''(z) < 0$. Thus, $U_i(x,z)$ is strictly concave in z. Also, by assumption, we have $W'(0) > V$, and $W'(1) < V$. Hence, z_u defined by Equation (E.2) is the unique global maximum.

Full compatibility (CC)

If application A_j is compatible with rival hardware H_i, where $j \neq i$, then we are in full compatibility regime CC. A multi-product user's utility is

$$U_i(x,z) = \begin{cases} (1-2z)V - p_1 + 2W(z) - r_1 - r_2 - tx, & \text{if she consumes } H_1, A_1 \text{ and } A_2 \ (i=1), \text{ and} \\ (1-2z)V - p_2 + 2W(z) - r_2 - r_1 - t(1-x), & \text{if she consumes } H_2, A_1 \text{ and } A_2 \ (i=2). \end{cases} \tag{E.3}$$

Differentiating $U_i(x,z)$ (defined by Equation (E.3)) with respect to z, yields a multi-product user's optimal choice z_u, where z_u is characterized by

$$\frac{\partial W(z_u)}{\partial z} = V, \in \{1,2\}. \tag{E.4}$$

Using the same argument as under regime NN, we can show that z_u defined by Equation (E.4) is the unique global maximum.

Asymmetric compatibility (NC)

Without loss of generality, suppose that application A_1 is incompatible with hardware H_2, whereas application A_2 is compatible with hardware H_1. Then we are in an asymmetric compatibility regime NC. A multi-product user's net utility is

$$U_i(x,z) = \begin{cases} (1-2z)V - p_1 + 2W(z) - r_1 - r_2 - tx, & \text{if she consumes } H_1, A_1 \text{ and } A_2 \ (i=1), \text{ and} \\ (1-z)V - p_2 + W(z) - r_2 - t(1-x), & \text{if she consumes } H_2 \text{ and } A_2 \ (i=2). \end{cases} \tag{E.5}$$

Differentiating $U_i(x,z)$ (defined by Equation (E.5)) with respect to z, yields a multi-product user's optimal choice z_u, where z_u is characterized by

$$\frac{\partial W(z_u)}{\partial z} = V, \in \{1,2\}. \tag{E.6}$$

Using the same argument as under regime NN, we can show that z_u defined by Equation (E.6) is the unique global maximum.

E.2 Stages 5, 4 and 3: Demand from single-product and multi-product users, and advertising demand functions

If a single-product user located at $x \in [0,1]$ purchases hardware H_i at price p_i and views a_i advertisements, then her net utility is as defined by Equation (2), yielding location of indifferent user $\hat{x}$ as defined by Equation (A.1), and thus demand functions remain the same as under the baseline model.

Next, depending on the compatibility regime, we can have three different scenarios for multi-product users.

Incompatibility (NN)

If application A_j is not compatible with rival hardware H_i, where $j \neq i$, then we are in an incompatible regime NN. A multi-product user's net utility obtained from consuming H_i, $i \in \{1,2\}$ is obtained by substituting $z = z_u$ in $U_i(x,z)$ (defined by Equation (E.1)), where z_u is defined by Equation (E.2). At *Stage 5*, substituting $z = z_u$ in Equation (10), yields the demand X_i for application A_i, where z_u is defined by Equation (E.2). At *Stage 4*, using Equation (E.1) (evaluated at $z = z_u$, where z_u is defined by Equation (E.2)) the location of indifferent multi-product user is defined by Equation (A.2) (evaluated at $z = z_u$, where z_u is defined by Equation (E.2)). This yields demand functions defined by Equation (A.3) (evaluated at $z = z_u$, where z_u is defined by Equation (E.2)). Therefore, the total demand for hardware H_1 and H_2 from both single-product and multi-product users is as defined by Equation (A.4) (evaluated at $z = z_u$, where z_u is defined by Equation (E.2)).

Full compatibility (CC)

If application A_j is compatible with rival hardware H_i, where $j \neq i$, then we are in full compatibility regime CC. A multi-product user's net utility from consuming H_i, $i \in \{1,2\}$ is obtained by substituting $z = z_u$ in $U_i(x,z)$ (defined by Equation (E.3)) at $z = z_u$, where z_u is defined by Equation (E.4). At *Stage 5*, substituting $z = z_u$ in Equation (13), yields the demand X_i for application A_i, where z_u is defined by Equation (E.4). At *Stage 4*, using Equation (E.3) (evaluated at $z = z_u$, where z_u is defined by Equation (E.4)), the location of indifferent multi-product user is defined by Equation (A.5) (evaluated at $z = z_u$, where z_u is defined by Equation (E.4)). This yields demand functions defined by Equation (A.6) (evaluated at $z = z_u$, where z_u

is defined by Equation (E.4)). Therefore, the total demand for hardware H_1 and H_2 from both single-product and multi-product users is as defined by Equation (A.7) (evaluated at $z = z_u$, where z_u is defined by Equation (E.4)).

Asymmetric compatibility (NC)

Without loss of generality, suppose that application A_1 is incompatible with hardware H_2, whereas application A_2 is compatible with hardware H_1. Then we are in an asymmetric compatibility regime NC. A multi-product user's net utility from consuming H_i, $i \in \{1, 2\}$ is obtained by substituting $z = z_u$ in $U_i(x, z)$ (defined by Equation (E.5)) at $z = z_u$, where z_u is defined by Equation (E.6). At *Stage 5*, substituting $z = z_u$ in Equation (16), yields the demand X_i for application A_i, where z_u is defined by Equation (E.6). At *Stage 4*, using Equation (E.5) (evaluated at $z = z_u$, where z_u is defined by Equation (E.6)), the location of indifferent multi-product user is defined by Equation (A.8) (evaluated at $z = z_u$, where z_u is defined by Equation (E.6)). This yields demand functions defined by Equation (A.9) (evaluated at $z = z_u$, where z_u is defined by Equation (E.6)). Therefore, the total demand for hardware H_1 and H_2 from both single-product and multi-product users is as defined by Equation (A.10) (evaluated at $z = z_u$, where z_u is defined by Equation (E.6)).

E.3 Stage 2: Equilibrium prices and demands under three distinct compatibility regimes

Using the demand functions for hardware, applications, and advertising (all evaluated at $z = z_u$), one can substitute $z = z_u$ into Lemmas 1, 2, and 3, where z_u is characterized in Lemma 4, to obtain the optimal prices, demands, and profits in each regime.

E.4 Stage 1: Equilibrium compatibility regime

We proceed through a series of steps. In *Step 1*, we characterize the conditions for the adoption of compatibility by the firms. In *Step 2*, we compare the thresholds obtained from *Step 1*. In *Step 3*, we characterize firms' optimal strategies.

Step 1: Conditions for choosing compatibility by the firms and comparison of thresholds.

Suppose firm $i \in \{1, 2\}$ believes that firm $j \neq i$ chooses incompatibility. Then, firm i is indifferent between choosing incompatibility and compatibility when compatibility cost is $F_1(\alpha)$, such that (i) $\pi_i^{NN}(F) = \pi_i^{CN}(F)$, at $F = F_1(\alpha)$, (ii) $\pi_i^{NN}(F) < \pi_i^{CN}(F)$, for all $F < F_1(\alpha)$, and (iii) $\pi_i^{NN}(F) > \pi_i^{CN}(F)$, for all $F > F_1(\alpha)$, where $F_1(\alpha)$ is obtained by substituting $z = z_u$ into Equation (B.29). In summary, $F_1(\alpha)$ is the threshold on compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application incompatible.

Suppose that firm $i \in \{1, 2\}$ believes that firm $j \neq i$ chooses compatibility. Then, firm i is indifferent between choosing incompatibility and compatibility when its compatibility cost is $F_2(\alpha)$, such that (i) $\pi_i^{NC}(F) = \pi_i^{CC}(F)$, at $F = F_2(\alpha)$, (ii) $\pi_i^{NC}(F) < \pi_i^{CC}(F)$, for all $F < F_2(\alpha)$, and (iii) $\pi_i^{NC}(F) > \pi_i^{CC}(F)$, for all $F > F_2(\alpha)$, where $F_2(\alpha)$ is as defined by substituting $z = z_u$ into Equation (B.30). In summary, $F_2(\alpha)$ is the threshold for compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application compatible.

Step 2: Comparison of thresholds $F_1(\alpha)$ and $F_2(\alpha)$.

Note that, at $\alpha = 0$, the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ take the same values, $F_1(0) = F_2(0) = 0$. Combining this with Claims 10, 11, and 12, we can show that $F_1(\alpha) \geq F_2(\alpha)$ for all $\alpha \in [0, 1]$, with strict inequality for $\alpha \in (0, 1]$.

Step 3: Optimal strategies and equilibrium outcomes.

Since, $F_1(\alpha) \geq F_2(\alpha)$ for all $\alpha \in [0, 1]$, in a market characterized by compatibility cost (F) and a fraction of multi-product users (α), the following holds.

(i) For a sufficiently small compatibility cost, i.e., $F < F_2(\alpha)$, both firms choose compatibility, i.e., regime CC, is the equilibrium outcome. Whereas, for a sufficiently large compatibility cost, i.e., $F \geq F_1(\alpha)$, both firms choose incompatibility, i.e., regime NN, is the equilibrium outcome.

(ii) Otherwise, there exist multiple equilibria with either firm 1 choosing incompatibility and firm 2 choosing compatibility, i.e., regime NC, or firm 1 choosing compatibility and firm 2 choosing incompatibility, i.e., regime CN, as the equilibrium outcome. ■

F Nuisance Costs

In this extension, we also consider the cost to users due to the presence of advertisements and derive, in addition to the equilibrium outcomes of the baseline model, the optimal level of advertisements. Let $\delta \geq 0$ denote the nuisance cost per advertisement (per unit of viewing time). To, bring out the main insights clearly, we will solve the model for $\delta = q$.[F.1] If a single-product user located at $x \in [0, 1]$ purchases hardware H_i at price p_i and views a_i advertisements, then her net utility is

$$U_i(x) = \begin{cases} V - p_1 - \delta a_1 - tx, & \text{if she consumes } H_1 \ (i = 1)\text{, and} \\ V - p_2 - \delta a_2 - t(1 - x), & \text{if she consumes } H_2 \ (i = 2). \end{cases} \tag{F.1}$$

A multi-product user's net utility depends on the compatibility regime.

Incompatibility (NN)

If application A_j is not compatible with rival hardware H_i, where $j \neq i$, then we are in an incompatible regime NN. A multi-product user's net utility is

$$U_i(x) = \begin{cases} (1 - z)V - p_1 - (1 - z)\delta a_1 + W(z) - r_1 - tx, & \text{if she consumes } H_1 \text{ and } A_1 \ (i = 1)\text{, and} \\ (1 - z)V - p_2 - (1 - z)\delta a_2 + W(z) - r_2 - t(1 - x), & \text{if she consumes } H_2 \text{ and } A_2 \ (i = 2). \end{cases} \tag{F.2}$$

If application A_j is compatible with rival hardware H_i, where $j \neq i$, then we are in full compatibility regime CC. A multi-product user's net utility is

$$U_i(x) = \begin{cases} (1 - 2z)V - p_1 - (1 - 2z)\delta a_1 + 2W(z) - r_1 - r_2 - tx, & \text{if she consumes } H_1, A_1, \\ & \text{and } A_2 \ (i = 1)\text{, and} \\ (1 - 2z)V - p_2 - (1 - 2z)\delta a_2 + 2W(z) - r_2 - r_1 - t(1 - x), & \text{if she consumes } H_2, A_1, \\ & \text{and } A_2 \ (i = 2). \end{cases} \tag{F.3}$$

[F.1] The main results hold for the case $q > \delta$.

If application A_1 is incompatible with hardware H_2, whereas application A_2 is compatible with hardware H_1, then we are in an asymmetric compatibility regime NC. A multi-product user's net utility is

$$U_i(x) = \begin{cases} (1-2z)V - p_1 - (1-2z)\delta a_1 + 2W(z) - r_1 - r_2 - tx, & \text{if she consumes } H_1, A_1, \\ & \text{and } A_2\ (i=1), \text{ and} \\ (1-z)V - p_2 - (1-z)\delta a_2 + W(z) - r_2 - t(1-x), & \text{if she consumes } H_2 \text{ and } A_2\ (i=2). \end{cases} \tag{F.4}$$

If application A_1 is compatible with hardware H_2, whereas application A_2 is incompatible with hardware H_1, then we are in an asymmetric compatibility regime CN. A multi-product user's net utility is

$$U_i(x) = \begin{cases} (1-z)V - (1-z)\delta a_1 - p_1 + W(z) - r_1 - tx, & \text{if she consumes } H_1 \text{ and } A_1\ (i=1), \text{ and} \\ (1-2z)V - (1-2z)\delta a_2 - p_2 + 2W(z) - r_1 - r_2 - t(1-x), & \text{if she consumes } H_2, A_1 \\ & \text{and } A_2\ (i=2). \end{cases} \tag{F.5}$$

F.1 Stages 5, 4 and 3: Demand from single-product and multi-product users, and advertising demand functions

Using Equation (F.1), the single-product user indifferent between hardware H_1 and H_2 is located at $\hat{x}$, such that (i) $U_1(\hat{x}) = U_2(\hat{x})$, (ii) $U_1(x) > U_2(x)$, for all $x < \hat{x}$, and (iii) $U_1(x) < U_2(x)$, for all $x > \hat{x}$, where

$$\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{\delta(a_2 - a_1)}{2t}. \tag{F.6}$$

Depending on the compatibility regime, we can have four different scenarios for multi-product users. **Incompatibility (NN)**

Using Equation (F.2), the multi-product user indifferent between hardware H_1 and H_2 is located at $\bar{x}$, such that (i) $U_1(\bar{x}) = U_2(\bar{x})$, (ii) $U_1(x) > U_2(x)$, for all $x < \bar{x}$, and (iii) $U_1(x) < U_2(x)$, for all $x > \bar{x}$, where

$$\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{r_2 - r_1}{2t} + \frac{(1-z)\delta(a_2 - a_1)}{2t}. \tag{F.7}$$

Using Equation (F.7), demand for hardware H_1 and H_2 from multi-product users is

$$\begin{aligned} D_1 &= \alpha\bar{x} = \alpha\left[\frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{r_2 - r_1}{2t} + \frac{(1-z)\delta(a_2 - a_1)}{2t}\right], \text{ and} \\ D_2 &= \alpha(1-\bar{x}) = \alpha\left[\frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{r_1 - r_2}{2t} + \frac{(1-z)\delta(a_1 - a_2)}{2t}\right]. \end{aligned} \tag{F.8}$$

Combining Equations (F.6) and (F.7) gives the total demand for hardware H_1 and H_2 (from both single-product and multi-product users) as

$$\begin{aligned} N_1 &= (1-\alpha)\hat{x} + \alpha\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{\alpha(r_2 - r_1)}{2t} + \frac{\delta(a_2 - a_1)}{2t} - \frac{\alpha z\delta(a_2 - a_1)}{2t}, \text{ and} \\ N_2 = 1 - N_1 &= \frac{1}{2} + \frac{p_1 - p_2}{2t} + \alpha\frac{(r_1 - r_2)}{2t} + \frac{\delta(a_1 - a_2)}{2t} - \frac{\alpha z\delta(a_1 - a_2)}{2t}. \end{aligned} \tag{F.9}$$

Full compatibility (CC)

Using Equation (F.3), the multi-product user indifferent between hardware H_1 and H_2 is located at $\bar{x}$, such that (i) $U_1(\bar{x}) = U_2(\bar{x})$, (ii) $U_1(x) > U_2(x)$, for all $x < \bar{x}$, and (iii) $U_1(x) < U_2(x)$, for all $x > \bar{x}$, where

$$\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{(1-2z)\delta(a_2 - a_1)}{2t}. \tag{F.10}$$

Using Equation (F.10) gives demand for hardware H_1 and H_2 from multi-product users as

$$\begin{aligned} D_1 &= \alpha\bar{x} = \alpha\left[\frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{(1-2z)\delta(a_2 - a_1)}{2t}\right], \text{ and} \\ D_2 &= \alpha(1-\bar{x}) = \alpha\left[\frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{(1-2z)\delta(a_1 - a_2)}{2t}\right]. \end{aligned} \tag{F.11}$$

Combining Equations (F.6) and (F.10) gives the total demand for hardware H_1 and H_2 (from both single-product and multi-product users) as

$$\begin{aligned} N_1 &= (1-\alpha)\hat{x} + \alpha\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{\delta(a_2 - a_1)}{2t} - \frac{2\alpha z\delta(a_2 - a_1)}{2t}, \text{ and} \\ N_2 &= 1 - N_1 = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\delta(a_1 - a_2)}{2t} - \frac{2\alpha z\delta(a_1 - a_2)}{2t}, \end{aligned} \tag{F.12}$$

Asymmetric compatibility (NC or CN)

Without loss of generality, suppose application A_1 is incompatible with hardware H_2, whereas application A_2 is compatible with hardware H_1. Using Equation (F.4), the multi-product user indifferent between hardware H_1 and H_2 is located at $\bar{x}$, such that (i) $U_1(\bar{x}) = U_2(\bar{x})$, (ii) $U_1(x) > U_2(x)$, for all $x < \bar{x}$, and (iii) $U_1(x) < U_2(x)$, for all $x > \bar{x}$, where

$$\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{r_1}{2t} + \frac{W(z)}{2t} - \frac{zV}{2t} + \frac{(1-z)\delta(a_2 - a_1)}{2t} + \frac{z\delta a_1}{2t}. \tag{F.13}$$

Using Equation (F.13), demand for hardware H_1 and H_2 from multi-product users is

$$\begin{aligned} D_1 &= \alpha\bar{x} = \alpha\left[\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{r_1}{2t} + \frac{W(z)}{2t} - \frac{z(V - \delta a_1)}{2t} + \frac{(1-z)\delta(a_2 - a_1)}{2t}\right], \text{ and} \\ D_2 &= \alpha(1-\bar{x}) = \alpha\left[\frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{r_1}{2t} - \frac{W(z)}{2t} + \frac{z(V - \delta a_1)}{2t} + \frac{(1-z)\delta(a_1 - a_2)}{2t}\right]. \end{aligned} \tag{F.14}$$

Combining Equations (F.6) and (F.13) gives the total demand for hardware H_1 and H_2 (from both single-product and multi-product users) as

$$\begin{aligned} N_1 &= (1-\alpha)\hat{x} + \alpha\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W(z)}{2t} - \frac{\alpha z(V - \delta a_1)}{2t} + \frac{\delta(a_2 - a_1)}{2t} - \frac{\alpha z\delta(a_2 - a_1)}{2t}, \text{ and} \\ N_2 &= 1 - N_1 = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha r_1}{2t} - \frac{\alpha W(z)}{2t} + \frac{\alpha z(V - \delta a_1)}{2t} + \frac{\delta(a_1 - a_2)}{2t} - \frac{\alpha z\delta(a_1 - a_2)}{2t}. \end{aligned} \tag{F.15}$$

This completes the derivation of demand functions of hardware and applications.

F.2 Stage 2: Equilibrium prices and demands under three distinct compatibility regimes

Incompatibility (NN)

Using the advertising demand function (defined by Equation (12)), we have

$$s_i = \frac{(1-z)qD_i + q(N_i - D_i)}{N_i}, \; i \in \{1,2\}. \tag{F.16}$$

The application demand function is defined by Equation (10). Since firm i will optimally choose $r_i \in [0, W(z)]$, we have $X_i = D_i$. Now, using $X_i = D_i$ and s_i (defined by Equation (F.16)) in Equation (1), the profit of firm $i \in \{1,2\}$ is

$$\pi_i \;=\; p_i N_i + r_i D_i + a_i(qN_i - zqD_i). \tag{F.17}$$

Substituting demands from Equations (F.8) and (F.9) in the preceding profit functions, the first-order conditions to firm i's profit maximization are

$$\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} \;&=\; \frac{1}{2} + \frac{p_j - p_i}{2t} + \alpha\frac{(r_j - r_i)}{2t} + \frac{\delta(a_j - a_i)}{2t} - \frac{\alpha z \delta(a_j - a_i)}{2t} \\
&\quad + p_i\left(\frac{-1}{2t}\right) + r_i\left(\frac{-\alpha}{2t}\right) + a_i\left[q\left(\frac{-1}{2t}\right) - zq\left(\frac{-\alpha}{2t}\right)\right] \;\leq\; 0, \qquad \text{(F.18)} \\
\frac{\partial \pi_i}{\partial r_i} \;&=\; p_i\left(\frac{-\alpha}{2t}\right) + \frac{\alpha}{2} + \alpha\frac{(p_j - p_i)}{2t} + \alpha\frac{r_j - r_i}{2t} + \alpha\frac{\delta(1-z)(a_j - a_i)}{2t} \\
&\quad + (r_i)\left(\frac{-\alpha}{2t}\right) + a_i\left(q\left(\frac{-\alpha}{2t}\right) - zq\left(\frac{-\alpha}{2t}\right)\right) \;\leq\; 0, \text{ and} \qquad \text{(F.19)} \\
\frac{\partial \pi_i}{\partial a_i} \;&=\; p_i\left(\frac{-\delta(1-\alpha z)}{2t}\right) + r_i\left(\frac{-\alpha\delta(1-z)}{2t}\right) + a_i\left[q\left(\frac{-\delta(1-\alpha z)}{2t}\right) - zq\left(\frac{-\alpha\delta(1-z)}{2t}\right)\right] \\
&\quad + q\left(\frac{1}{2} + \frac{p_j - p_i}{2t} + \alpha\frac{(r_j - r_i)}{2t} + \frac{\delta(a_j - a_i)}{2t} - \frac{\alpha z\delta(a_j - a_i)}{2t}\right) \\
&\quad - zq\left(\frac{\alpha}{2} + \alpha\frac{(p_j - p_i)}{2t} + \alpha\frac{r_j - r_i}{2t} + \alpha\frac{\delta(1-z)(a_j - a_i)}{2t}\right) \;\leq\; 0, \qquad \text{(F.20)}
\end{aligned}$$

where subscript $j \neq i$ denotes the rival firm. Using Equations (F.18) and (F.19), setting $\frac{\partial \pi_i}{\partial p_i} = 0$, $\frac{\partial \pi_i}{\partial r_i} = 0$, and $\frac{\partial \pi_i}{\partial a_i} = 0$, and imposing symmetry (since firms have symmetric objective functions), we obtain the symmetric optimal hardware prices as

$$p_1^{NN} = p_2^{NN} = t - a^{NN}q, \tag{F.21}$$

which are non-negative, given the assumption of non-negative prices (Assumption 2), and symmetric optimal application prices are

$$r_1^{NN} = r_2^{NN} = a^{NN}zq. \tag{F.22}$$

Since each firmâ€™s profit function π_i is strictly concave in its own choice variables: p_i, r_i and a_i (holding the rivalâ€™s prices and advertising level fixed), and also, the symmetric solution to the firstâ€‘order conditions is strictly interior (i.e. positive), given the assumption of non-negative prices (Assumption 2), the interior solution constitutes the unique global maximum.

The equilibrium demand for hardware H_i is $N_i = \frac{1}{2}, i \in \{1,2\}$, the equilibrium demand for application A_i is $X_i = D_i = \frac{\alpha}{2},\ i \in \{1,2\}$ (because $W(z) > r^{NN} = a^{NN}zq$), and the equilibrium advertising level is $a_1^{NN} = a_2^{NN} \in (0,1]$, defined by

$$p^{NN}\left(\frac{-\delta(1-\alpha z)}{2t}\right) + r^{NN}\left(\frac{-\alpha\delta(1-z)}{2t}\right) + a^{NN}\left[q\left(\frac{-\delta(1-\alpha z)}{2t}\right) - zq\left(\frac{-\alpha\delta(1-z)}{2t}\right)\right] + \left[q\left(\frac{1}{2}\right) - zq\left(\frac{\alpha}{2}\right)\right] = 0$$

Combining this with Equation (F.16), we have

$$s_1^{NN} = s_2^{NN} = q - \alpha zq. \tag{F.23}$$

Using Equation (F.17), firms' profit are $\pi_1^{NN} = \pi_2^{NN} = \frac{t}{2}$.

Full Compatibility

At *Stage 3*, using the advertising demand function (defined by Equation (15)), we have

$$s_i = \frac{(1-2z)qD_i + q(N_i - D_i)}{N_i},\ i \in \{1,2\}. \tag{F.24}$$

Also, optimal r_i must lie between $[0, W(z)]$. Therefore, using the application demand function (defined by Equation (13)), we have that $X_i = D_i + D_j = \alpha$. Now, using these values in Equation (1), the profit of firm $i \in \{1, 2\}$ is

$$\pi_i \;=\; p_i N_i + r_i \alpha + a_i(qN_i - 2zqD_i) - F. \tag{F.25}$$

Substituting demands from Equations (F.11) and (F.12) in the preceding profit functions, the first-order conditions to firm i's profit maximization are

$$\begin{aligned}\frac{\partial \pi_i}{\partial p_i} \;&=\; \frac{1}{2} + \frac{p_j - p_i}{2t} + \frac{\delta(a_j - a_i)}{2t} - \frac{2\alpha z\delta(a_j - a_i)}{2t} \\ &\qquad + p_i\left(\frac{-1}{2t}\right) + r_i\left(\frac{-\alpha}{2t}\right) + a_i\left[q\left(\frac{-1}{2t}\right) - 2zq\left(\frac{-\alpha}{2t}\right)\right] \;\le\; 0,\end{aligned} \tag{F.26}$$

$$\frac{\partial \pi_i}{\partial r_i} \;=\; \alpha \;>\; 0, \text{ and} \tag{F.27}$$

$$\begin{aligned}\frac{\partial \pi_i}{\partial a_i} \;&=\; p_i\left(\frac{-\delta(1-2\alpha z)}{2t}\right) + r_i\left(\frac{-\alpha\delta(1-2z)}{2t}\right) + a_i\left[q\left(\frac{-\delta(1-2\alpha z)}{2t}\right) - 2zq\left(\frac{-\alpha\delta(1-2z)}{2t}\right)\right] \\ &\quad + q\left(\frac{1}{2} + \frac{p_j - p_i}{2t} + \frac{\delta(a_j - a_i)}{2t} - \frac{2\alpha z\delta(a_j - a_i)}{2t}\right) - 2zq\left(\frac{\alpha}{2} + \alpha\frac{(p_j - p_i)}{2t} + \alpha\frac{\delta(1-2z)(a_j - a_i)}{2t}\right) \;\le\; 0,\end{aligned} \tag{F.28}$$

where subscript $j \neq i$ denotes the rival firm. Using Equations (F.26) and (F.28), setting $\frac{\partial \pi_i}{\partial p_i} = 0$, and $\frac{\partial \pi_i}{\partial a_i} = 0$ and imposing symmetry (since firms have symmetric objective functions), we obtain the symmetric optimal hardware prices as

$$p_1^{CC} = p_2^{CC} = t - a^{CC}q + 2\alpha zqa^{CC}, \tag{F.29}$$

which are non-negative, given the assumption of non-negative prices (Assumption 2). Since each firmâ€™s profit function π_i is strictly concave in its own choice variables: p_i and a_i (holding the rivalâ€™s price and advertising level fixed), and also, the symmetric solution to the firstâ€‘order conditions is strictly interior (i.e. positive), given the assumption of non-negative prices (Assumption 2), the interior solution constitutes the unique global maximum.

Equation (F.27) shows that the profits are strictly increasing in application prices. Thus, using the application demand function (defined by Equation (13)), we have

$$r_1^{CC} = r_2^{CC} = W(z). \tag{F.30}$$

The equilibrium demand for hardware H_i is $N_i = \frac{1}{2}, i \in \{1,2\}$, the equilibrium demand for application A_i is $X_i = \alpha,\ i \in \{1,2\}$, and the equilibrium demand for advertising is $a_1^{CC} = a_2^{CC} \in [0,1]$, defined by defined by

$$(p^{CC} + q)\left(\frac{-\delta(1-2\alpha z)}{2t}\right) + r^{CC}\left(\frac{-\alpha\delta(1-2z)}{2t}\right) + a^{CC}\left[q\left(\frac{-\delta(1-2\alpha z)}{2t}\right) - 2zq\left(\frac{-\alpha\delta(1-2z)}{2t}\right)\right]$$
$$+ \left[q\left(\frac{1}{2}\right) - 2zq\left(\frac{\alpha}{2}\right)\right] = 0$$

Using Equation (F.24), we have

$$s_1^{CC} = s_2^{CC} = q - 2\alpha z q \tag{F.31}$$

Using Equation (F.25), firms' profit are $\pi_1^{CC} = \pi_2^{CC} = \frac{t}{2} + \alpha W(z) - F$.

Asymmetric Compatibility

Without loss of generality, suppose that firm 1 chooses to make its application A_1 incompatible with rival firmâ€™s hardware H_2, whereas firm 2 chooses to make its application A_2 compatible with rival firmâ€™s hardware H_1. At *Stage 3*, using the advertising demand function (defined by Equation (18)), we have

$$s_1 = \frac{(1-2z)qD_1 + q(N_1 - D_1)}{N_1}, \text{ and } s_2 = \frac{(1-z)qD_2 + q(N_2 - D_2)}{N_2}. \tag{F.32}$$

Second, any optimal r_i must be between $[0, W(z)]$, using the application demand function (defined by Equation (16)), we have that $X_1 = D_1$ and $X_2 = D_1 + D_2 = \alpha$. Now, using these values in Equation (1), the profit of firms are

$$\pi_1 = p_1 N_1 + r_1 D_1 + a_1(qN_1 - 2zqD_1) \text{ and } \pi_2 = p_2 N_2 + r_2\alpha + a_2(qN_2 - zqD_2) - F. \tag{F.33}$$

Substituting demand from Equations (F.14) and (F.15) in the preceding profit functions, under the condition $\delta = q$, and after algebraic calculations, the first-order conditions to firm 1 and firm 2's profit maximization are

$$\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} + \frac{\alpha(W(z) - zV)}{2t} + \frac{1}{2t}\left[p_2 - 2p_1 - 2\delta(1-2\alpha z)a_1 + \delta(1-\alpha z)a_2 - 2\alpha r_1\right] \leq 0, \quad \text{(F.34)}$$

$$\frac{\partial \pi_1}{\partial r_1} = \frac{\alpha}{2} + \frac{\alpha(W(z) - zV)}{2t} + \frac{\alpha}{2t}\left[p_2 - 2p_1 - 2\delta(1-2z)a_1 + \delta(1-z)a_2 - 2r_1\right] \leq 0, \quad \text{(F.35)}$$

$$\frac{\partial \pi_1}{\partial a_1} = \frac{\delta}{2t}[t - 2\alpha tz + \alpha(1-2z)(W(z) - zV) - 2\delta(1 - 4\alpha z(1-z)a_1) + \delta(1 - \alpha z(3-2z))a_2$$

$$-2p_1 + p_2 + 2\alpha(2zp_1 - zp_2 - (1-2z)r_1)] \leq 0, \quad \text{(F.36)}$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} + \frac{1}{2t}[\alpha(zV - W(z)) + \delta(1-2\alpha z)a_1 - 2\delta(1-\alpha z)a_2 + p_1 - 2p_2 + ar_1] \leq 0, \quad \text{(F.37)}$$

$$\frac{\partial \pi_1}{\partial r_1} = \alpha > 0, \quad \text{(F.38)}$$

$$\frac{\partial \pi_2}{\partial a_2} = \frac{\delta}{2t}[t - \alpha tz - \alpha(1-z)(W(z) - zV) + \delta(1 - \alpha z(3-2z))a_1 -$$

$$2\delta(1 - \alpha(2-z)z)a_2 + p_1 - 2p_2 + \alpha(r_1 - z(p_1 - 2p_2 + r_1))] \leq 0,. \quad \text{(F.39)}$$

Using first-order conditions defined by Equations (F.34)- (F.39), setting $\frac{\partial \pi_i}{\partial p_i} = 0$, $\frac{\partial \pi_1}{\partial r_1} = 0$, and $\frac{\partial \pi_i}{\partial a_i} = 0$,, and solving, we obtain the optimal hardware H_1 and application prices and advertising levels. They are

$$p_1^{NC} = \frac{1}{6}[6t + \alpha Vz - \alpha W(Z) - 6\delta a_1^{NC} + 3\alpha\delta z a_2^{NC}], \; p_2^{NC} = \frac{1}{3}[3t + \alpha Vz - \alpha W(z) - 3\delta a_2^{NC} + 3a\delta z a_2^{NC}]; \quad \text{(F.40)}$$

$$r_1^{NC} = \frac{1}{2}[zV - W(z) + 4z\delta a_1^{NC} - \delta z a_2^{NC}], \; r_2^{NC} = W(z); \quad \text{(F.41)}$$

$$a_1^{NC} = 0 \text{ and } a_2^{NC} = \frac{W(z) - zV}{3\delta}. \quad \text{(F.42)}$$

Note that hardware and application prices are non-negative, given the assumption of full market coverage (Assumption 1) and non-negative prices (Assumption 2).

The equilibrium demand is obtained by substituting Equations (F.40) and (F.41) in Equations

(F.14) and (F.15) to get

$$N_1^{NC} = \frac{1}{2}+\frac{\alpha(W(z)-zV)}{6t}, \; N_2^{NC} = 1-N_2^{NC} \; X_1^{NC} = D_1^{NC} = \frac{\alpha}{2}+\frac{\alpha(W(z)-zV)}{6t}, \;\; X_2^{NC} = D_1+D_2 = \alpha, \tag{F.43}$$

and the equilibrium advertising prices are

$$s_1^{NC} = \frac{(1-2z)qD_1^{NC} + q(N_1^{NC} - D_1^{NC})}{N_1^{NC}}, \; and \; s_2 = \frac{(1-z)qD_2^{NC} + q(N_2^{NC} - D_2^{NC})}{N_2^{NC}}. \tag{F.44}$$

Using Equation (F.33), firms' profit are $\pi_1^{NC} = p_1^{NC}N_1^{NC} + r_1^{NC}D_1^{NC}$, and $\pi_2^{NC} = p_2^{NC}N_2^{NC} + r_2^{NC}\alpha + a_2^{NC}[qN_2^{NC} - zqD_2^{NC}] - F$.

F.3 Stage 1: Equilibrium compatibility regime

We proceed through a series of steps. In *Step 1*, we characterize the conditions for the adoption of compatibility by the firms. In *Step 2*, we compare the thresholds obtained from *Step 1*. In *Step 3*, we characterize firms' optimal strategies.

Step 1: Conditions for choosing compatibility by the firms and comparison of thresholds.

Suppose firm $i \in \{1,2\}$ believes that firm $j \neq i$ chooses incompatibility. Then, firm i is indifferent between choosing incompatibility and compatibility when compatibility cost is $F_1(\alpha)$, such that (i) $\pi_i^{NN}(F) = \pi_i^{CN}(F)$, at $F = F_1(\alpha)$, (ii) $\pi_i^{NN}(F) < \pi_i^{CN}(F)$, for all $F < F_1(\alpha)$, and (iii) $\pi_i^{NN}(F) > \pi_i^{CN}(F)$, for all $F > F_1(\alpha)$, where $F_1(\alpha)$ is obtained by solving

$$\frac{t}{2} = p_2^{NC}N_2^{NC} + r_2^{NC}\alpha + a_2^{NC}[qN_2^{NC} - zqD_2^{NC}] - F_1(\alpha).$$

Using values of p_2^{NC} (defined by Equation (F.40)), r_2^{NC} (defined by Equation (F.41)), a_2^{NC} (defined by Equation (F.42)), N_2^{NC}, and $D_2^{NC} = \alpha - D_1^{NC}$ (defined by Equation (F.43)), and after algebraic calculations, we obtain

$$F_1(\alpha) = \frac{1}{18t}[9t^2 + 6\alpha tz(zV + 2W(z)) + \alpha(zV - W(z))^2] - \frac{t}{2}. \tag{F.45}$$

In summary, $F_1(\alpha)$ is the threshold for compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application incompatible.

Suppose that firm $i \in \{1,2\}$ believes that firm $j \neq i$ chooses compatibility. Then, firm i is indifferent between choosing incompatibility and compatibility when its compatibility cost is $F_2(\alpha)$, such that (i) $\pi_i^{NC}(F) = \pi_i^{CC}(F)$, at $F = F_2(\alpha)$, (ii) $\pi_i^{NC}(F) < \pi_i^{CC}(F)$, for all $F < F_2(\alpha)$, and (iii) $\pi_i^{NC}(F) > \pi_i^{CC}(F)$, for all $F > F_2(\alpha)$, where $F_2(\alpha)$ is obtained by solving

$$\frac{t}{2} + \alpha W(z) - F_2(\alpha) = p_1^{NC} N_1^{NC} + r_1^{NC} D_1^{NC}.$$

Using values of p_1^{NC} (defined by Equation (F.40)), r_1^{NC} (defined by Equation (F.41)), a_1^{NC} (defined by Equation (F.42)), N_1^{NC} and D_1^{NC} (defined by Equation (F.43)), and after algebraic calculations, we obtain

$$F_2(\alpha) = \frac{t}{2} + \alpha W(z) - \frac{1}{18t}[9t^2 - 6\alpha t(zV - W(z)) + \alpha(zV - W(z))^2]. \tag{F.46}$$

In summary, $F_2(\alpha)$ is the threshold for compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application compatible.

Step 2: Comparison of thresholds $F_1(\alpha)$ and $F_2(\alpha)$.

Claim 16. $\frac{\partial F_1(\alpha)}{\partial \alpha} > 0$, *for all* $\alpha > 0$.

Proof. Using Equation (F.45), and differentiating $F_1(\alpha)$ with respect to α, and after algebraic calculations we obtain

$$\frac{\partial F_1(\alpha)}{\partial \alpha} = \frac{1}{18}[(6t(zV + 2W(z)) + (zV - W(z))^2)] > 0. \tag{F.47}$$

Hence, we have $\frac{\partial F_1(\alpha)}{\partial \alpha} > 0$, for all $\alpha > 0$. ■

Claim 17. $\frac{\partial F_2(\alpha)}{\partial \alpha} > 0$, *for all* $\alpha > 0$.

Proof. Using Equation (F.46), and differentiating $F_2(\alpha)$ with respect to α, and after algebraic calculations we obtain

$$\frac{\partial F_2(\alpha)}{\partial \alpha} = \frac{1}{18}[(6t(zV + 2W(z)) - (zV - W(z))^2)] > 0, \tag{F.48}$$

because $t \geq \frac{(zV-W(z))^2}{zV+2W(z)}$. Hence, we have $\frac{\partial F_2(\alpha)}{\partial \alpha} > 0$, for all $\alpha > 0$. ■

Claim 18. $\frac{\partial F_1(\alpha)}{\partial \alpha} > \frac{\partial F_2(\alpha)}{\partial \alpha}$, *for all* $\alpha > 0$.

Proof. Using Equations (F.47) and (F.48), we have

$$\frac{\partial F_1(\alpha)}{\partial \alpha} - \frac{\partial F_2(\alpha)}{\partial \alpha} = \frac{1}{9t}[zV - W(z)]^2 > 0.$$

Finally, note that, at $\alpha = 0$, the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ take the same values, $F_1(0) = F_2(0) = 0$. Combining this with Claims 16, 17, and 18, we have $F_1(\alpha) \geq F_2(\alpha)$ for all $\alpha \in [0, 1]$.

Step 3: Optimal strategies and equilibrium outcomes.

Since, $F_1(\alpha) \geq F_2(\alpha)$ for all $\alpha \in [0, 1]$, in a market characterized by compatibility cost (F) and a fraction of multi-product users (α), the following holds.

(i) For a sufficiently small compatibility cost, i.e., $F < F_2(\alpha)$, both firms choose compatibility, i.e., regime CC, is the equilibrium outcome. Whereas, for a sufficiently large compatibility cost, i.e., $F \geq F_1(\alpha)$, both firms choose incompatibility, i.e., regime NN, is the equilibrium outcome.

(ii) Otherwise, there exist multiple equilibria with either firm 1 choosing incompatibility and firm 2 choosing compatibility, i.e., regime NC, or firm 1 choosing compatibility and firm 2 choosing incompatibility, i.e., regime CN, as the equilibrium outcome.

G Heterogeneous Advertisers

In this extension, we allow for heterogeneity in advertisers' revenue from advertising on hardware and also derive the optimal advertising levels. Let θq denote the advertiser's revenue per unit of time spent on a hardware per user by placing an advertisement on hardware H_i, where θ is uniformly distributed over [0, 1]. If application A_j is not compatible with rival hardware H_i, where $j \neq i$, then we are in an incompatible regime NN. The profit of advertiser θ is

$$\theta[(1 - z)qD_i + q(N_i - D_i)] - s_i N_i. \tag{G.1}$$

If application A_j is compatible with rival hardware H_i, where $j \neq i$, then we are in full compatibility regime CC. The profit of advertiser θ from placing an advertisement in hardware H_i, $i \in \{1, 2\}$ is

$$\theta[(1-2z)qD_i + q(N_i - D_i)] - s_i N_i. \tag{G.2}$$

If application A_i is incompatible with hardware H_j, whereas application A_j is compatible with hardware H_i, then we are in an asymmetric compatibility regime. The profit of advertiser θ from placing an advertisement in hardware H_i, $i \in \{1, 2\}$ is

$$\begin{cases} \theta[(1-2z)qD_i + q(N_i - D_i)] - s_i N_i, & \text{from placing an ad in hardware } H_i \text{, and} \\ \theta[(1-z)qD_j + q(N_j - D_j)] - s_j N_j, & \text{from placing an ad in hardware } H_j. \end{cases} \tag{G.3}$$

G.1 Stages 5, 4 and 3: Demand from single-product and multi-product users, and advertising demand functions

If a single-product user located at $x \in [0, 1]$ purchases hardware H_i at price p_i and views a_i advertisements, then her net utility is as defined by Equation (2), yielding location of indifferent user $\hat{x}$ as defined by Equation (A.1) under the baseline model.

Depending on the compatibility regime, we can have four different scenarios for multi-product users.

Incompatibility (NN)

If application A_j is not compatible with rival hardware H_i, where $j \neq i$, then we are in an incompatible regime NN. At *Stage 5*, we obtain demand X_i for application A_i as defined by Equation (10). At *Stage 4*, a multi-product user's net utility is as defined by Equation (3), yielding location of indifferent multi-product user as defined by Equation (A.2), and demand functions remains the same as under the baseline model (defined by Equation (A.3)). Therefore, the total demand functions for hardware H_1 and H_2 from both single-product and multi-product users remains the same as under baseline model (defined by Equation (A.4)). At *Stage 3*, given advertising price s_i, $i \in \{1, 2\}$, an advertiser θ places an advertisement in hardware H_i, $i \in \{1, 2\}$

if $\theta[(1-z)qD_i+q(N_i-D_i)]-s_iN_i \geq 0$. The marginal advertiser $\hat{\theta}_i$ indifferent between advertising and not advertising in hardware H_i, $i \in \{1,2\}$ is $\hat{\theta}_i = \frac{s_iN_i}{(1-z)qD_i+q(N_i-D_i)}$. Using it, the level of advertisements in hardware H_i, $i \in \{1,2\}$ is $a_i = 1-\hat{\theta}_i = 1-\frac{s_iN_i}{(1-z)qD_i+q(N_i-D_i)}$. This gives the inverse advertising demand function of hardware H_i, $i \in \{1,2\}$ as

$$s_i = (1-a_i)\frac{[(1-z)qD_i+q(N_i-D_i)]}{N_i}. \tag{G.4}$$

Full compatibility (CC)

If application A_j is compatible with rival hardware H_i, where $j \neq i$, then we are in full compatibility regime CC. At *Stage 5*, we obtain demand X_i for application A_i as defined by Equation (13). At *Stage 4*, a multi-product user's net utility is as defined by Equation (4), yielding location of indifferent multi-product user as defined by Equation (A.5), and demand functions remains the same as under the baseline model (defined by Equation (A.6)). Therefore, the total demand functions for hardware H_1 and H_2 from both single-product and multi-product users remains the same as under baseline model (defined by Equation (A.7)).

At *Stage 3*, given advertising price s_i, $i \in \{1,2\}$, an advertiser θ places an advertisement in hardware $H_i, i \in \{1,2\}$ if $\theta[(1-2z)qD_i+q(N_i-D_i)]-s_iN_i \geq 0$. The marginal advertiser $\hat{\theta}_i$ indifferent between advertising and not advertising in hardware H_i, $i \in \{1,2\}$ is $\hat{\theta}_i = \frac{s_iN_i}{(1-2z)qD_i+q(N_i-D_i)}$. Using it, the level of advertisements in hardware H_i, $i \in \{1,2\}$ is $a_i = 1-\hat{\theta}_i = 1-\frac{s_iN_i}{(1-2z)qD_i+q(N_i-D_i)}$ This gives the inverse advertising demand function of hardware $H_i, i \in \{1,2\}$ as

$$s_i = (1-a_i)\frac{[(1-2z)qD_i+q(N_i-D_i)]}{N_i}. \tag{G.5}$$

Asymmetric compatibility (NC)

Without loss of generality, suppose that application A_1 is incompatible with hardware H_2, whereas application A_2 is compatible with hardware H_1. Then, we are in an asymmetric compatibility regime NC. At *Stage 5*, we obtain demand X_i for application A_i as defined by Equation (16). At *Stage 4*, a multi-product user's net utility is as defined by Equation (5), yielding location of indifferent multi-product user as defined by Equation (A.8), and demand functions remains the same as under the baseline model (defined by Equation (A.9)). Therefore, the total demand functions for hardware H_1 and H_2 from both single-product and multi-product users remains the same as under baseline model (defined by Equation (A.10)). At *Stage 3*, given advertising price s_i, $i \in \{1, 2\}$, an advertiser θ places an advertisement in hardware H_1 if $\theta[(1-2z)qD_1 + q(N_1 - D_1)] - s_1 N_1 \geq 0$. The marginal advertiser $\hat{\theta}_1$ indifferent between advertising and not advertising in hardware H_1, is $\hat{\theta}_1 = \frac{s_1 N_1}{(1-2z)qD_1 + q(N_1 - D_1)}$. Using it, the level of advertisements in hardware H_1 is $a_1 = 1 - \hat{\theta}_1 = 1 - \frac{s_1 N_1}{(1-2z)qD_1 + q(N_1 - D_1)}$. This gives the inverse advertising demand function of hardware H_1, as

$$s_1 = (1 - a_1)\frac{[(1-2z)qD_1 + q(N_2 - D_1)]}{N_1}. \tag{G.6}$$

Similarly, an advertiser θ places an advertisement in hardware H_2 if $\theta[(1-z)qD_2 + q(N_2 - D_2)] - s_2 N_2 \geq 0$. The marginal advertiser $\hat{\theta}_2$ indifferent between advertising and not advertising in hardware H_2, is $\hat{\theta}_2 = \frac{s_2 N_2}{(1-z)qD_2 + q(N_2 - D_2)}$. Using it, the level of advertisements in hardware H_2 is $a_2 = 1 - \hat{\theta}_2 = 1 - \frac{s_2 N_2}{(1-z)qD_2 + q(N_2 - D_2)}$. This gives the inverse advertising demand function of hardware H_2, as

$$s_2 = (1 - a_2)\frac{[(1-z)qD_2 + q(N_2 - D_2)]}{N_2}. \tag{G.7}$$

G.2 Stage 2: Equilibrium prices and demands under three distinct compatibility regimes

Incompatibility (NN)

The application demand function is defined by Equation (10). Since firm i will optimally choose $r_i \in [0, W(z)]$, we have $X_i = D_i$. Now, using the inverse advertising demand function (defined by Equation (G.4)) in Equation (1), the profit of firm $i \in \{1, 2\}$ is

$$\pi_i = p_i N_i + r_i D_i + a_i(1 - a_i)(qN_i - zqD_i). \tag{G.8}$$

Substituting demands from Equations (A.3) and (A.4) in the preceding profit functions, the

first-order conditions to firm i's profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{2} + \frac{p_j - p_i}{2t} + \alpha \frac{(r_j - r_i)}{2t} + +p_i \left(\frac{-1}{2t}\right)$$
$$+ r_i \left(\frac{-\alpha}{2t}\right) + a_i(1 - a_i) \left[q\left(\frac{-1}{2t}\right) - zq\left(\frac{-\alpha}{2t}\right)\right] \leq 0, \tag{G.9}$$
$$\frac{\partial \pi_i}{\partial r_i} = p_i \left(\frac{-\alpha}{2t}\right) + \frac{\alpha}{2} + \alpha \frac{(p_j - p_i)}{2t} + \alpha \frac{r_j - r_i}{2t}$$
$$+ r_i \left(\frac{-\alpha}{2t}\right) + a_i(1 - a_i)\left(q\left(\frac{-\alpha}{2t}\right) - zq\left(\frac{-\alpha}{2t}\right)\right) \leq 0, \text{ and} \tag{G.10}$$
$$\frac{\partial \pi_i}{\partial a_i} = (1 - 2a_i)\left[q\left(\frac{1}{2} + \frac{p_j - p_i}{2t} + \alpha \frac{(r_j - r_i)}{2t}\right)\right]$$
$$- (1 - 2a_i)\left[zq\left(\frac{\alpha}{2} + \alpha \frac{(p_j - p_i)}{2t} + \alpha \frac{r_j - r_i}{2t}\right)\right] \leq 0, \tag{G.11}$$

where subscript $j \neq i$ denotes the rival firm. Using Equations (G.9), (G.10), (G.11), setting $\frac{\partial \pi_i}{\partial p_i} = 0$, $\frac{\partial \pi_i}{\partial r_i} = 0$, and $\frac{\partial \pi_i}{\partial a_i} = 0$ and imposing symmetry (since firms have symmetric objective functions), we obtain the symmetric optimal hardware and application prices, and advertising levels as

$$p_1^{NN} = p_2^{NN} = t - \frac{q}{4}, \; r_1^{NN} = r_2^{NN} = \frac{zq}{4}, \text{ and } a_1^{NN} = a_2^{NN} = \frac{1}{2}. \tag{G.12}$$

Prices are non-negative, given the assumption of non-negative prices (Assumption 2). Since each firmâ€™s profit function π_i is strictly concave in its own choice variables: p_i, r_i and a_i (holding the rivalâ€™s prices and advertising level fixed), and also, the symmetric solution to the firstâ€'order conditions is strictly interior (i.e. positive), given the assumption of non-negative prices (Assumption 2)), the interior solution constitutes the unique global maximum.

The equilibrium demand for hardware H_i is $N_i = \frac{1}{2}, i \in \{1, 2\}$, the equilibrium demand for application A_i is $X_i = D_i = \frac{\alpha}{2}, \; i \in \{1, 2\}$ (because $W(z) > r^{NN}$). Therefore, Using Equation (G.8), firms' profit are $\pi_1^{NN} = \pi_2^{NN} = \frac{t}{2}$.

Full Compatibility

The application demand function is defined by Equation (13). Since firm i will optimally choose $r_i \in [0, W(z)]$, we have $X_i = D_i = D_j = \alpha$. Now, using the inverse advertising demand function (defined by Equation (G.5)) in Equation (1), the profit of firm $i \in \{1, 2\}$ is

$$\pi_i \;=\; p_i N_i + r_i \alpha + a_i(1-a_i)(qN_i - 2zqD_i) - F. \tag{G.13}$$

Substituting demands from Equations (A.6) and (A.7) in the preceding profit function, the first-order conditions to firm i's profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} \;=\; \frac{1}{2} + \frac{p_j - p_i}{2t} + p_i\left(\frac{-1}{2t}\right) + a_i(1-a_i)\left[q(1-2\alpha z)\left(\frac{-1}{2t}\right)\right] \;\le\; 0, \tag{G.14}$$

$$\frac{\partial \pi_i}{\partial r_i} \;=\; \alpha > 0, \text{ and} \tag{G.15}$$

$$\frac{\partial \pi_i}{\partial a_i} \;=\; (1-2a_i)\left[q(1-2\alpha z)\left(\frac{1}{2} + \frac{p_j - p_i}{2t}\right)\right] \;\le\; 0, \tag{G.16}$$

where subscript $j \neq i$ denotes the rival firm. Since $\frac{\partial \pi_i}{\partial r_i} > 0$, firm i will charge $r_i = W(z)$. Using Equations (G.14), (G.15), (G.16), setting $\frac{\partial \pi_i}{\partial p_i} = 0$, and $\frac{\partial \pi_i}{\partial a_i} = 0$ and imposing symmetry (since firms have symmetric objective functions), we obtain the symmetric optimal hardware and application prices, and advertising levels as

$$p_1^{CC} = p_2^{CC} = t - \frac{q}{4} + \frac{\alpha z q}{2},\; r_1^{CC} = r_2^{CC} = W(z), \text{ and } a_1^{CC} = a_2^{CC} = \frac{1}{2}. \tag{G.17}$$

Prices are non-negative, given the assumption of non-negative prices (Assumption 2). Since each firmâ€™s profit function π_i is strictly concave in its own choice variables: p_i, and a_i (holding the rivalâ€™s price and advertising level fixed), and also, the symmetric solution to the firstâ€'order conditions is strictly interior (i.e. positive), given the assumption of non-negative prices (Assumption 2), the interior solution constitutes the unique global maximum.

The equilibrium demand for hardware H_i is $N_i = \frac{1}{2}, i \in \{1, 2\}$, the equilibrium demand for application A_i is $X_i = \alpha,\; i \in \{1, 2\}$ (because $W(z) > r^{NN}$). Therefore, using Equation (G.13), firms' profit are $\pi_1^{CC} \;=\; \pi_2^{CC} \;=\; \frac{t}{2} + \alpha W(z) - F$.

Asymmetric Compatibility

Without loss of generality, suppose that firm 1 chooses to make its application A_1 incompatible with rival firmâ€™s hardware H_2, whereas firm 2 chooses to make its application A_2 compatible with rival firmâ€™s hardware H_1. Second, any optimal r_i must be between $[0, W(z)]$, using the application demand function (defined by Equation (16)), we have $X_1 = D_1$ and $X_2 = D_1 + D_2 = \alpha$. Now, using these values and inverse advertising demand function in Equation (1), the profit of the firms are

$$\pi_1 \;=\; p_1N_1 + r_1D_1 + a_1(1-a_1)(qN_1 - 2zqD_1), \text{and } \pi_2 \;=\; p_2N_2 + r_2\alpha + a_2(1-a_2)(qN_2 - zqD_2) - F. \tag{G.18}$$

Substituting demands from Equations (A.9) and (A.10) in the preceding profit function, the first-order conditions to firm 1 and firm 2's profit maximization are

$$\frac{\partial \pi_1}{\partial p_1} \;=\; \frac{1}{8t}[4t + q(-1 + 2\alpha z) - 4\alpha(zV - W(z)) - 8p_1 + 4p_2 - 8\alpha r_1] \;\leq\; 0, \tag{G.19}$$

$$\frac{\partial \pi_1}{\partial r_1} \;=\; \frac{\alpha}{8t}[4t + q(-1 + 2z) - 4(zV - W(z)) - 8p_1 + 4p_2 - 8r_1] \;\leq\; 0, \tag{G.20}$$

$$\begin{aligned}\frac{\partial \pi_1}{\partial a_1} \;=\;& (1 - 2a_1)\left[q\left(\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W(z)}{2t} - \frac{\alpha z V}{2t}\right)\right] \\ & - (1 - 2a_1)\left[2\alpha z q\left(\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{r_1}{2t} + \frac{W(z)}{2t} - \frac{zV}{2t}\right)\right] \leq\; 0,\end{aligned} \tag{G.21}$$

$$\frac{\partial \pi_2}{\partial p_2} \;=\; \frac{1}{8t}[q(-1 + \alpha z) + 4(t + \alpha(zV - W(z)) + 4p_1 - 8p_2 + 4\alpha r_1] \;\leq\; 0, \tag{G.22}$$

$$\frac{\partial \pi_2}{\partial r_2} \;=\; \alpha > 0, \tag{G.23}$$

$$\begin{aligned}\frac{\partial \pi_2}{\partial a_2} \;=\;& (1 - 2a_2)\left[q\left(\frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha r_1}{2t} - \frac{\alpha W(z)}{2t} + \frac{\alpha z V}{2t}\right)\right] \\ & - (1 - 2a_2)zq\alpha\left[\frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{r_1}{2t} - \frac{W(z)}{2t} + \frac{zV}{2t}\right] \;\leq\; 0,.\end{aligned} \tag{G.24}$$

Using first-order conditions defined by Equations (G.19) - (G.24), setting $\frac{\partial \pi_i}{\partial p_i} = 0$, $\frac{\partial \pi_1}{\partial r_1} = 0$, and $\frac{\partial \pi_i}{\partial a_i} = 0$,, and solving, we obtain the optimal hardware and application prices and advertising levels. They are

$$p_1^{NC} \;=\; \frac{1}{12}[-3q + 12t + 2\alpha qz + 2\alpha Vz - 2\alpha W(z)] \text{ and } p_2^{NC} \;=\; \frac{1}{12}[-3q + 12t + 4\alpha qz + 4\alpha Vz - 4\alpha W(z)] \tag{G.25}$$

$$r_1^{NC} \;=\; \frac{1}{4}[qz - 2Vz + 2W(z)] \text{ and } r_2^{NC} \;= W(z), \text{ and } a_1^{NC} = a_2^{NC} \;= \frac{1}{2}. \tag{G.26}$$

Note that hardware price are non-negative, given the assumption of full market coverage (Assumption 1) and non-negative prices (Assumption 2).

Substituting equilibrium prices and advertising levels in Equations (A.9) and (A.10), the equilibrium demands are

$$N_1^{NC} = \frac{1}{2} - \frac{[\alpha(zq + 4(zV - W(z)))]}{24t}, \; N_2^{NC} = 1 - N_2^{NC},$$
$$X_1^{NC} = D_1^{NC} = \frac{\alpha}{2} + \frac{\alpha[(-3 + 2\alpha)qz + 2(-3 + \alpha)(zV - W(z))]}{24t}, \; X_2^{NC} = D_1 + D_2 = \alpha. \quad \text{(G.27)}$$

Firms' profit are $\pi_1^{NC} = p_1^{NC} N_1^{NC} + r_1^{NC} D_1^{NC} + a_1^{NC}(1 - a_1^{NC})(qN_1^{NC} - 2zqD_1^{NC})$, and $\pi_2^{NC} = p_2^{NC} N_2^{NC} + r_2^{NC}\alpha + a_2^{NC}[qN_2^{NC} - zqD_2^{NC}] - F$.

G.3 Stage 1: Equilibrium compatibility regime

We proceed through a series of steps. In *Step 1*, we characterize the conditions for the adoption of compatibility by the firms. In *Step 2*, we compare the thresholds obtained from *Step 1*. In *Step 3*, we characterize firms' optimal strategies.

Step 1: Conditions for choosing compatibility by the firms and comparison of thresholds.

Suppose firm $i \in \{1, 2\}$ believes that firm $j \neq i$ chooses incompatibility. Then, firm i is indifferent between choosing incompatibility and compatibility when compatibility cost is $F_1(\alpha)$, such that (i) $\pi_i^{NN}(F) = \pi_i^{CN}(F)$, at $F = F_1(\alpha)$, (ii) $\pi_i^{NN}(F) < \pi_i^{CN}(F)$, for all $F < F_1(\alpha)$, and (iii) $\pi_i^{NN}(F) > \pi_i^{CN}(F)$, for all $F > F_1(\alpha)$, where $F_1(\alpha)$ is obtained by solving

$$\frac{t}{2} = p_2^{NC} N_2^{NC} + r_2^{NC}\alpha + a_2^{NC}[qN_2^{NC} - zqD_2^{NC}] - F_1(\alpha).$$

Using values of p_2^{NC} (defined by Equation (G.25)), r_2^{NC} and a_2^{NC} (defined by Equation (G.26)), N_2^{NC} and $D_2^{NC} = \alpha - D_1^{NC}$ (defined by Equation (G.27)), and after algebraic calculations, we obtain

$$F_1(\alpha) = \frac{1}{288}\left[144t + 24\alpha(zq + 4zV + 8W(z))\right]$$
$$+ \frac{1}{288t}[\alpha(z^2(-9 + 10\alpha)q^2 + 2(-9 + 13\alpha)qz(zV - W(z)) + 16\alpha(zV - W(z))^2)] - \frac{t}{2}. \quad \text{(G.28)}$$

In summary, $F_1(\alpha)$ is the threshold for compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application incompatible.

Suppose that firm $i \in \{1, 2\}$ believes that firm $j \neq i$ chooses compatibility. Then, firm i is indifferent between choosing incompatibility and compatibility when its compatibility cost is $F_2(\alpha)$, such that (i) $\pi_i^{NC}(F) = \pi_i^{CC}(F)$, at $F = F_2(\alpha)$, (ii) $\pi_i^{NC}(F) < \pi_i^{CC}(F)$, for all $F < F_2(\alpha)$, and (iii) $\pi_i^{NC}(F) > \pi_i^{CC}(F)$, for all $F > F_2(\alpha)$, where $F_2(\alpha)$ is obtained by solving

$$\frac{t}{2} + \alpha W(z) - F_2(\alpha) = p_1^{NC} N_1^{NC} + r_1^{NC} D_1^{NC} + a_1^{NC}(1 - a_1^{NC})(q N_1^{NC} - 2zq D_1^{NC}).$$

Using values of p_1^{NC} (defined by Equation (G.25)), r_1^{NC} and a_1^{NC} defined by Equation (G.26)), N_1^{NC} and D_1^{NC} (defined by Equation (G.27)), and after algebraic calculations, we obtain

$$\begin{aligned} F_2(\alpha) = {} & \frac{t}{2} + W(z) - \frac{1}{288}\left[144t - 24\alpha(zq + 4(zV - W(z))\right] \\ & - \frac{1}{288t}[\alpha(z^2(9 - 8\alpha)q^2 - 4z(-9 + 7\alpha)q(zV - W(z)) - 4(-9 + 5\alpha)(zV - W(z))^2)]. \end{aligned} \tag{G.29}$$

In summary, $F_2(\alpha)$ is the threshold for compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application compatible.

Step 2: Comparison of thresholds $F_1(\alpha)$ and $F_2(\alpha)$.

Claim 19. $\frac{\partial F_1(\alpha)}{\partial \alpha} > 0$, *for all* $\alpha > 0$.

Proof. Using Equation (G.28), and differentiating $F_1(\alpha)$ with respect to α, and after algebraic calculations we obtain

$$\begin{aligned} \frac{\partial F_1(\alpha)}{\partial \alpha} = {} & \frac{1}{288t}[24zqt + (-9 + 20\alpha)q^2 z^2 + 96t(zV + 2W(z))] \\ & + \frac{1}{288t}[2(-9 + 26\alpha)qz(zV - W(z)) + 32\alpha(zV - W(z))^2] > 0. \end{aligned} \tag{G.30}$$

Hence, we have $\frac{\partial F_1(\alpha)}{\partial \alpha} > 0$, for all $\alpha > 0$. ∎

Claim 20. $\frac{\partial F_2(\alpha)}{\partial \alpha} > 0$, *for all* $\alpha > 0$.

Proof. Using Equation (G.29), and differentiating $F_2(\alpha)$ with respect to α, and after algebraic calculations, we obtain

$$\frac{\partial F_2(\alpha)}{\partial \alpha} = \frac{1}{288t}[(-9+16\alpha)q^2z^2 + 96t(zV + 2W(z))] + \frac{1}{288t}[4(-9+10\alpha)(zV - W(z))^2 + 4zq(6t + (-9+14\alpha)(zV - W(z)))] > 0. \tag{G.31}$$

Hence, we have $\frac{\partial F_2(\alpha)}{\partial \alpha} > 0$, for all $\alpha > 0$. ■

Claim 21. $\frac{\partial F_1(\alpha)}{\partial \alpha} > \frac{\partial F_2(\alpha)}{\partial \alpha}$, *for all* $\alpha > 0$.

Proof. Using Equations (G.30) and (G.31), we have

$$\frac{\partial F_1(\alpha)}{\partial \alpha} - \frac{\partial F_2(\alpha)}{\partial \alpha} = \frac{1}{144t}[2\alpha(zq + zV - W(z))(zq - 2(zV - W(z)))] + \frac{1}{144t}[9(zV - W(z))(zq + 2(zV - W(z)))]$$

$$\frac{\partial F_1(\alpha)}{\partial \alpha} - \frac{\partial F_2(\alpha)}{\partial \alpha} = \frac{1}{144t}[2\alpha(zV - W(z))(-2(zV - W(z)))] + \frac{1}{144t}[9(zV - W(z))(2(zV - W(z)))]$$
$$= \frac{1}{144t}[W(z) - zV]^2(18 - 4\alpha) > 0. \tag{G.32}$$

Finally, note that, at $\alpha = 0$, the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ take the same values, $F_1(0) = F_2(0) = 0$. Combining this with Claims 19, 20, and 21, we have $F_1(\alpha) \geq F_2(\alpha)$ for all $\alpha \in [0, 1]$.

Step 3: Optimal strategies and equilibrium outcomes.

Since, $F_1(\alpha) \geq F_2(\alpha)$ for all $\alpha \in [0, 1]$, in a market characterized by compatibility cost (F) and a fraction of multi-product users (α), the following holds.

(i) For a sufficiently small compatibility cost, i.e., $F < F_2(\alpha)$, both firms choose compatibility, i.e., regime CC, is the equilibrium outcome. Whereas, for a sufficiently large compatibility cost, i.e., $F \geq F_1(\alpha)$, both firms choose incompatibility, i.e., regime NN, is the equilibrium outcome.

(ii) Otherwise, there exist multiple equilibria with either firm 1 choosing incompatibility and firm 2 choosing compatibility, i.e., regime NC, or firm 1 choosing compatibility and firm 2 choosing incompatibility, i.e., regime CN, as the equilibrium outcome.